Test exam

Introduction to Logic
Minor Logic and Computation

Exercise 1 (10 points)**.** Argue by making use of a truth table whether the following argument is valid or not. If it is not valid specify a counter-example.

$$(p \wedge q) \to r, \neg q \vee r, p/\neg q$$

Exercise 2 (10 points)**.** Prove that $\varphi \to \neg\psi$ is a contradiction iff φ and ψ are both tautologies.

Exercise 3 (15 points)**.** Translate the following sentences in the language of first-order predicate logic. Use the identity sign if necessary.

(1) All students who passed the exam are pleased with themselves.

(2) All students made at least two exams in this semester.

(3) John passed only one exam.

Exercise 4 (15 points)**.** Consider the model $\mathcal{M} = \langle D, I \rangle$, where

$$D = \{1, 2, 3, 4\}$$
$$I(R) = \{\langle 1, 2\rangle, \langle 1, 3\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle, \langle 3, 4\rangle\}$$
$$I(a) = 1; I(b) = 2; I(c) = 3; I(d) = 4$$

Argue whether the following sentences are true in this model or not.

1. $\forall y(\exists x Rxy \leftrightarrow \exists z Ryz)$
2. $\forall x \forall y \forall z((Rxy \wedge Ryz) \to Rxz)$
3. $\forall x \forall y(x = y \leftrightarrow (Rxy \leftrightarrow Ryx))$

Exercise 5 (20 points)**.** One of the following arguments is valid, the other is invalid.

$$\forall x(Ax \vee Bx),\ \forall x(Ax \to Cx),\ \exists x \neg Cx \ /\ \exists x Bx$$
$$\forall x \exists y \exists z(x \neq y \wedge x \neq z \wedge y \neq z \wedge Rxy \wedge Rxz)\ /\ \forall x \forall y(x \neq y \to (Rxy \vee Ryx))$$

If the argument is not valid, argue this fact by using a counter-model. If the argument is valid, give a proof.

Exercise 6 (20 points)**.** Show by means of a natural deduction that the following assertions are correct:

1. $(p \vee (q \wedge r)) \vdash (p \vee r)$
2. $\vdash (p \vee \neg p)$
3. $\neg \exists x(Fx \wedge Gx) \vdash \forall x(Fx \rightarrow \neg Gx)$
4. $\exists x Ax \rightarrow \forall x Bx, \exists x \neg Bx \vdash \neg \exists x Ax$
5. $(\exists x Rax \wedge \forall x Rxa) \rightarrow \forall x Rax, \forall x Rxa \vdash Rab$

Exercise 7 (10 points)**.** Consider the following properties of relations.

- Transitivity (TR),
- Antisymmetry (AS),
- Irreflexivity (IR).

Exactly one of these properties follows logically from the other two combined. This means that if a relation has two of these properties, it has the third one as well.

Which of the properties follows from the other two? Motivate your answer with an argument.