

Test exam

Introduction to Logic Minor Logic and Computation

Exercise 1 (10 points). Argue by making use of a truth table whether the following argument is valid or not. If it is not valid specify a counter-example.

$$(p \wedge q) \rightarrow r, \neg q \vee r, p / \neg q$$

Exercise 2 (10 points). Prove that $\varphi \rightarrow \neg\psi$ is a contradiction iff φ and ψ are both tautologies.

Exercise 3 (15 points). Translate the following sentences in the language of first-order predicate logic. Use the identity sign if necessary.

- (1) All students who passed the exam are pleased with themselves.
- (2) All students made at least two exams in this semester.
- (3) There is one exam that only John passed.

Exercise 4 (15 points). Consider the model $\mathcal{M} = \langle D, I \rangle$, where

$$\begin{aligned} D &= \{1, 2, 3, 4\} \\ I(R) &= \{\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 3, 4 \rangle\} \\ I(a) &= 1; I(b) = 2; I(c) = 3; I(d) = 4 \end{aligned}$$

Argue whether the following sentences are true in this model or not.

1. $\forall y(\exists x Rxy \leftrightarrow \exists z Ryz)$
2. $\forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz)$
3. $\forall x \forall y (x = y \leftrightarrow (Rxy \leftrightarrow Ryx))$

Exercise 5 (20 points). One of the following arguments is valid, the other is invalid.

$$\begin{aligned} &\forall x(Ax \vee Bx), \forall x(Ax \rightarrow Cx), \exists x \neg Cx / \exists x Bx \\ &\forall x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge Rxy \wedge Rxz) / \forall x \forall y (x \neq y \rightarrow (Rxy \vee Ryx)) \end{aligned}$$

If the argument is not valid, argue this fact by using a counter-model. If the argument is valid, give a proof.

Exercise 6 (20 points). Show by means of a natural deduction that the following assertions are correct:

1. $(p \vee (q \wedge r)) \vdash (p \vee r)$
2. $\vdash (p \vee \neg p)$
3. $\neg \exists x(Fx \wedge Gx) \vdash \forall x(Fx \rightarrow \neg Gx)$
4. $\exists xAx \rightarrow \forall xBx, \exists x\neg Bx \vdash \neg \exists xAx$
5. $(\exists xRax \wedge \forall xRxa) \rightarrow \forall xRax, \forall xRxa \vdash Rab$

Exercise 7 (10 points). Consider the following properties of relations.

- Transitivity (TR),
- Antisymmetry (AS),
- Irreflexivity (IR).

Exactly one of these properties follows logically from the other two combined. This means that if a relation has two of these properties, it has the third one as well.

Which of the properties follows from the other two? Motivate your answer with an argument.