Neglect-zero effects at the semantics-pragmatics interface

Maria Aloni
ILLC & Philosophy
University of Amsterdam
M.D.Aloni@uva.nl

UMass Amherst Linguistics Colloquium
6 October 2023
Nothing is logical (Nihil)

- **Goal of the project:** a formal account of a class of natural language inferences which deviate from classical logic
- **Common assumption:** these deviations are not logical mistakes, but rather consequence of pragmatic enrichment
- **Strategy:** develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- **Novel hypothesis:** *neglect-zero* tendency as crucial pragmatic factor
- **Main conclusion:** deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Nihil website
https://projects.illc.uva.nl/nihil/

Nihil team
MA, Anttila, Brinck Knudstorp, Degano, Klochowicz & Ramotowska (+ more collaborators including Sbardolini)
Non-classical inferences

Free choice (FC)

(1) $\diamond (\alpha \lor \beta) \leadsto \diamond \alpha \land \diamond \beta$

(2) Deontic FC inference
   a. You may go to the beach or to the cinema.
   b. $\leadsto$ You may go to the beach and you may go to the cinema.

(3) Epistemic FC inference
   a. Mr. X might be in Victoria or in Brixton.
   b. $\leadsto$ Mr. X might be in Victoria and he might be in Brixton.

Ignorance

(4) The prize is in the attic or in the garden $\leadsto$ speaker doesn’t know which
[Grice 1989]

(5) ? I have two or three children.

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC analysed as conversational implicatures; grammatical implicatures; semantic entailments; ...
Novel hypothesis: neglect-zero

- **FC** and ignorance inferences are \[\neq\] semantic entailments
- Not the result of Gricean reasoning \[\neq\] conversational implicatures
- Not the effect of applications of covert grammatical operators \[\neq\] scalar implicatures

- But rather a consequence of something else speakers do in conversation, namely,

**Neglect-Zero**

when interpreting a sentence speakers create structures representing reality\(^1\) and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (**zero-models**)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets \[\text{[Nieder 2016, Bott et al, 2019]}\]

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.
   a. Verifier: [■, ■, ■]
   b. Falsifier: [■, □, ■]
   c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]

(7) Less than three squares are black.
   a. Verifier: [■, □, ■]
   b. Falsifier: [■, ■, ■]
   c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]; [□, □, □]

Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
   the special status of 0 among the natural numbers [Nieder, 2016]
   why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (less vs more) [Bott et al., 2019]
   existential import & other principles operative in Aristotelian logic (every A is B ⇒ some A is B; not (if A then not A)) [MA, 2023]

Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences
Novel hypothesis: neglect-zero

Illustrations

(8) It is raining.
   a. Verifier: [***]
   b. Falsifier: [☼☼]
   c. Zero-models: none

(9) It is snowing.
   a. Verifier: [❄❄❄❄❄]
   b. Falsifier: [☼☼☼]; [***]; ... 
   c. Zero-models: none

(10) It is raining or snowing.
   a. Verifier: [*** | ❄❄❄❄❄]
   b. Falsifier: [☼☼☼]
   c. Zero-models: [*** ]; [❄❄❄❄❄]

▶ Two models in (10-c) are zero-models because they verify the sentence by virtue of an empty witness for one of the disjuncts
▶ Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded
Comparison with competing accounts

<table>
<thead>
<tr>
<th>Neo-Gricean Grammatical view</th>
<th>Ignorance inference</th>
<th>FC inference</th>
<th>Scalar implicature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nihil</td>
<td>reasoning</td>
<td>reasoning</td>
<td>reasoning</td>
</tr>
<tr>
<td></td>
<td>debated</td>
<td>grammatical</td>
<td>grammatical</td>
</tr>
<tr>
<td></td>
<td>neglect-zero</td>
<td>neglect-zero</td>
<td>—</td>
</tr>
</tbody>
</table>

Ignorance, free choice and scalar implicatures

➤ Scalar implicatures compatible with ignorance and free choice:

(11) Pat ate the cake or the ice-cream ⇐
    a. Speaker doesn’t know which (ignorance)
    b. P didn’t eat both (scalar implicature)

(12) Pat may eat the cake or the ice-cream ⇐
    a. Pat may choose which $\Diamond \alpha \land \Diamond \beta$ (free choice)
    b. Pat may not eat both $\neg \Diamond (\alpha \land \beta)$ (scalar implicature)

➤ Ignorance and free choice are incompatible

(13) Pat may eat the cake or the ice-cream, I don’t know which
    $\not\Rightarrow$ P may choose which (free choice cancellation)
BSML: teams and bilateralism

- **Team semantics:** formulas interpreted wrt a set of points of evaluation (a team) rather than single ones  
  [Väänänen 2007; Yang & Väänänen 2017]

Classical vs team-based modal logic

- Classical modal logic:  
  \[ M, w \models \phi, \text{ where } w \in W \]

- Team-based modal logic:  
  \[ M, t \models \phi, \text{ where } t \subseteq W \]

Bilateral state-based modal logic (BSML)

- Teams \(\mapsto\) information states  
  [Dekker93; Groenendijk+96; Ciardelli+19]

- Assertion & rejection conditions modeled rather than truth  
  [Anderson & Belnap75; Rumfitt00]

  \[ M, s \models \phi, \text{ “} \phi \text{ is assertable in } s \text{”}, \text{ with } s \subseteq W \]

  \[ M, s \models \phi, \text{ “} \phi \text{ is rejectable in } s \text{”}, \text{ with } s \subseteq W \]

- In BSML inferences relate speech acts rather than propositions and therefore might diverge from classical semantic entailments
Neglect-zero effects in BSML: split disjunction

- A state $s$ supports a disjunction $\phi \lor \psi$ iff $s$ is the union of two substates, each supporting one of the disjuncts.

![Diagram](image)

(a) No-zero verifier  
(b) Zero-model  
(c) Falsifier

**Figure:** Models for $(a \lor b)$.

- $\{w_a\}$ verifies $(a \lor b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$  
  [\mp{} zero-model]

- **Main idea:** define neglect-zero enrichments, $[\_]^+$, whose core effect is to rule out such zero-models.

- **Implementation:** $[\_]^+$ defined using $\text{NE} (s \models \text{NE} \text{ iff } s \neq \emptyset)$, which models neglect-zero in the logic.
Neglect-zero effects in BSML: enriched disjunction

- $s$ supports an enriched disjunction $[\phi \lor \psi]^+$ iff $s$ is the union of two non-empty substates, each supporting one of the disjuncts.

\begin{align*}
\text{(a)} & : [a \lor b]^+ \\
\text{(b)} & : \not\models [a \lor b]^+ \\
\text{(c)} & : \models [a \lor b]^+
\end{align*}

- An enriched disjunction requires both disjuncts to be live possibilities.

\begin{align*}
(14) & \quad \text{It is raining or snowing} \sim \text{It might be raining and it might be snowing} \\
& \quad \text{(epistemic) possibility}
\end{align*}

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to positive disjunctions.

\begin{align*}
\iff & \quad \text{we derive } FC \text{ and related effects (for pragmatically enriched formulas);} \\
\iff & \quad \text{pragmatic enrichment vacuous under single negation.}
\end{align*}
Neglect-zero effects in BSML: possibility vs uncertainty

▸ More no-zero verifiers for \( a \lor b \):  

\[
\begin{align*}
(w_{ab}, w_a) & \models [a \lor b]^+ \\
(w_{ab}, w_a, w_b) & \models [a \lor b]^+ \\
(w_{ab}, w_a, w_b) & \models [a \lor b]^+
\end{align*}
\]

▸ Two components of full ignorance (‘speaker doesn’t know which’):  

\[
\begin{align*}
\text{(Degano et al., 2023)} \\
(15) \quad \text{It is raining or it is snowing} \ (\alpha \lor \beta) \Rightarrow \\
\text{a. Uncertainty:} \quad & \neg \Box_e \alpha \land \neg \Box_e \beta \\
\text{b. Possibility:} \quad & \Diamond_e \alpha \land \Diamond_e \beta \quad \text{(equiv} \neg \Box_e \neg \alpha \land \neg \Box_e \neg \beta \text{)}
\end{align*}
\]

▸ Only possibility derived as neglect-zero effect:  

▸ \( \{w_{ab}, w_a\} \models \Diamond_e a \land \Diamond_e b \), but \( \not\models \neg \Box_e a \land \not\models \neg (a \land b) \)  

▸ \( \{w_{ab}, w_a\} \): a no-zero model supporting possibility but neither uncertainty nor scalar implicature  

[no-zero non-scalar verifier]
Two derivations of full ignorance

1. Neo-Gricean derivation
   
   (i) Uncertainty derived through *quantity* reasoning
   
   \[
   \alpha \lor \beta \tag{16}
   \]
   ASSERTION

   \[
   \neg \Box_e \alpha \land \neg \Box_e \beta \tag{17}
   \]
   UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and *quality* about assertion
   
   \[
   \Box_e (\alpha \lor \beta) \tag{18}
   \]
   QUALITY ABOUT ASSERTION

   \[
   \Rightarrow \Diamond_e \alpha \land \Diamond_e \beta \tag{19}
   \]
   POSSIBILITY

2. Nihil derivation

   (i) Possibility derived as *neglect-zero* effect

   \[
   \alpha \lor \beta \tag{20}
   \]
   ASSERTION

   \[
   \Diamond_e \alpha \land \Diamond_e \beta \tag{21}
   \]
   POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and *scalar* reasoning

   \[
   \neg (\alpha \land \beta) \tag{22}
   \]
   SCALAR IMPLICATURE

   \[
   \Rightarrow \neg \Box_e \alpha \land \neg \Box_e \beta \tag{23}
   \]
   UNCERTAINTY
Novel hypothesis: neglect-zero

Comparison with competing accounts

<table>
<thead>
<tr>
<th></th>
<th>Ignorance inference</th>
<th>FC inference</th>
<th>Scalar implicature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neo-Gricean</td>
<td>reasoning</td>
<td>reasoning</td>
<td>reasoning</td>
</tr>
<tr>
<td>Grammatical view</td>
<td>debated</td>
<td>grammatical</td>
<td>grammatical</td>
</tr>
<tr>
<td>Nihil</td>
<td>neglect-zero</td>
<td>neglect-zero</td>
<td>—</td>
</tr>
</tbody>
</table>

▶ **Ignorance**: Neo-Gricean vs Nihil predictions
  ▶ **Neo-Gricean**: No possibility without uncertainty
  ▶ **Nihil**: Possibility derived independently from uncertainty

Argument 1 in favor of neglect-zero

▶ Experimental findings in agreement with Nihil predictions\(^2\)
  ▶ Using adapted mystery box paradigm, compared conditions in which
    ▶ both uncertainty and possibility are false \([\text{zero-model}]\)
    ▶ uncertainty false but possibility true \([\text{no-zero non-scalar model}]\)
  ▶ Less acceptance when possibility is false (95% vs 44%)
  ▶ Evidence that possibility can arise without uncertainty

Novel hypothesis: neglect-zero

Comparison with competing accounts

<table>
<thead>
<tr>
<th></th>
<th>Ignorance inference</th>
<th>FC inference</th>
<th>Scalar implicature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neo-Gricean</td>
<td>reasoning</td>
<td>reasoning</td>
<td>reasoning</td>
</tr>
<tr>
<td>Grammatical view</td>
<td>debated</td>
<td>grammatical</td>
<td>grammatical</td>
</tr>
<tr>
<td>Nihil</td>
<td>neglect-zero</td>
<td>neglect-zero</td>
<td>—</td>
</tr>
</tbody>
</table>

Argument 2 in favor of neglect-zero

- **Cognitive plausibility:** differences between FC and scalar implicatures [Chemla & Bott, 2014; Tieu et al, 2016]:

<table>
<thead>
<tr>
<th>FC inference</th>
<th>processing cost</th>
<th>acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>early</td>
<td></td>
</tr>
<tr>
<td>high</td>
<td>late</td>
<td></td>
</tr>
</tbody>
</table>

- **Possible explanation for neo-Gricean or grammatical view:**
  - Scalar alternatives less accessible [Singh et al, 2016]

- **Still low cost and early acquisition of FC**
  - Hard to explain on neo-Gricean or grammatical view
  - Expected on neglect-zero hypothesis:
    - FC inference follows from the assumption that when interpreting sentences language users neglect zero-models
    - Zero-models neglected because cognitively taxing
What about scalar implicatures?

- No evidence of scalar implicatures in Degano et al (2023) experiment (sentence-picture verification task)
- Verification tasks arguably test production (speaker-oriented) rather than interpretation (hearer-oriented) (Degen & Goodman, 2014)
- Conjecture:
  - Production (speaker-oriented): only neglect-zero operative
  - Interpretation (hearer-oriented): neglect-zero + scalar reasoning
- First try: Neglect-zero + neo-Gricean strategies (Gazdar 1979)

<table>
<thead>
<tr>
<th>Possibility</th>
<th>FC inference</th>
<th>Scalar implicature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nihil 1</td>
<td>neglect-zero</td>
<td>neglect-zero</td>
</tr>
</tbody>
</table>

- But lack of explanation for following cases:

  (24) Mary is working at her paper or seeing some of her students ~ Mary is working at her paper or seeing not all of her students (Chierchia 2004)

  (25) Mary read some or all of the books (Chierchia et al, 2012)

  (26) Jane came or Jane and Maria came ~ Jane alone or Jane & Maria
What about scalar implicatures?

- **Second try**: Neglect-zero + local **exh** (or **pex**, Del Pinal *et al.*, 2021):

<table>
<thead>
<tr>
<th>Possibility</th>
<th>FC inference</th>
<th>Scalar implicature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nihil 2</td>
<td>neglect-zero</td>
<td>neglect-zero</td>
</tr>
</tbody>
</table>

- A simpler notion than ordinary **exh** (Fox 2007):

  (27) \[\text{exh}(\phi) = \phi \land \neg \alpha, \text{ for each relevant } \phi \text{-alternative } \alpha \text{ s.t. } \neg \alpha \] (contextually) consistent with \(\phi\)

  No reference to IE (innocent exclusion): \(\neg a\) consistent with \(a \lor b\), but inconsistent with \([a \lor b]^+\), since \([a \lor b]^+ \models \Diamond_a e\)

- **Other possible advantages**:
  - Possibly simplified theory of \(\phi\)-alternatives (only scalar and focal)
  - No need of recursive **exh** (or **pex***) for **FC**

- **Back to our questions**: (i) Why no evidence of scalar implicatures in our experiment? (ii) Why scalar implicatures more costly than **FC**?

- **Possible answers**:
  - Neglect-zero: constant pragmatic-cognitive factor, can be suspended but at a cost;
  - **exh/pex**: optional grammatical device, can be suspended at zero cost if disambiguation problem is resolved by context (verification task), otherwise it normally applies but with additional disambiguation costs.
Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

<table>
<thead>
<tr>
<th></th>
<th>NS FC</th>
<th>Dual Prohib</th>
<th>Universal FC</th>
<th>Double Neg</th>
<th>WS FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neo-Gricean</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>?</td>
<td>no</td>
</tr>
<tr>
<td>Grammatical</td>
<td>yes</td>
<td>yes*</td>
<td>yes</td>
<td>no*</td>
<td>no*</td>
</tr>
<tr>
<td>Semantic</td>
<td>yes</td>
<td>no*</td>
<td>yes</td>
<td>no*</td>
<td>no</td>
</tr>
<tr>
<td>Neglect-zero</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Argument 3 in favor of neglect-zero hypothesis

- **Empirical coverage:** FC sentences give rise to a complex pattern of inferences

(28)  
\[ \Diamond (\alpha \lor \beta) \sim \Diamond \alpha \land \Diamond \beta \]  
\[ \neg \Diamond (\alpha \lor \beta) \sim \neg \Diamond \alpha \land \neg \Diamond \beta \]  
\[ \forall x (\alpha \lor \beta) \sim \forall x (\Diamond \alpha \land \Diamond \beta) \]  
\[ \neg \neg \Diamond (\alpha \lor \beta) \sim \Diamond \alpha \land \Diamond \beta \]  
\[ \Diamond \alpha \lor \Diamond \beta \sim \Diamond \alpha \land \Diamond \beta \]  

- Captured by neglect-zero approach implemented in BSML\(^3\)
- Most other approaches need additional assumptions

---

The data

(29) **Dual Prohibition** [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream.
   \[\sim \alpha \vee \beta \sim \neg \Diamond \alpha \land \neg \Diamond \beta \]

b. \[\neg \Diamond (\alpha \vee \beta) \sim \neg \Diamond \alpha \land \neg \Diamond \beta \]

(30) **Universal FC** [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
   \[\sim \forall x (\Diamond (\alpha \vee \beta)) \sim \forall x (\Diamond \alpha \land \Diamond \beta) \]

b. \[\forall x (\Diamond (\alpha \vee \beta)) \sim \forall x (\Diamond \alpha \land \Diamond \beta) \]

(31) **Double Negation FC** [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus.
   \[\sim \exists x (\neg \Diamond (\alpha(x) \vee \beta(x))) \land \forall y (y \neq x \rightarrow \neg \Diamond (\alpha(y) \vee \beta(y))) \]

b. \[\exists x (\neg \Diamond (\alpha(x) \vee \beta(x))) \land \forall y (y \neq x \rightarrow (\Diamond \alpha(y) \land \Diamond \beta(y))) \]

(32) **Wide Scope FC** [Zimmermann 2000, Hoeks *et al.* 2017]

a. Detectives may go by bus or they may go by boat.
   \[\sim \Diamond \alpha \lor \Diamond \beta \sim \Diamond \alpha \land \Diamond \beta \]

b. Mr. X might be in Victoria or he might be in Brixton.
   \[\sim \Diamond \alpha \lor \Diamond \beta \sim \Diamond \alpha \land \Diamond \beta \]

b. Mr. X might be in Victoria and might be in Brixton.
Neglect-zero effects in BSML: predictions

After pragmatic enrichment

- We derive both wide and narrow scope FC inferences:
  - Narrow scope FC: $[\diamond (\alpha \lor \beta)]^+_C \models \diamond \alpha \land \diamond \beta$
  - Universal FC: $[\forall x \diamond (\alpha \lor \beta)]^+_C \models \forall x (\diamond \alpha \land \diamond \beta)$
  - Double negation FC: $[\neg \neg \diamond (\alpha \lor \beta)]^+_C \models \diamond \alpha \land \diamond \beta$
  - Wide scope FC: $[\diamond \alpha \lor \diamond \beta]^+_C \models \diamond \alpha \land \diamond \beta$ (if $R$ is indisputable)

- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[-\diamond (\alpha \lor \beta)]^+_C \models -\diamond \alpha \land -\diamond \beta$

Before pragmatic enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

  $$\alpha \models_{BSML^0} \beta \iff \alpha \models_{CML} \beta \quad [\alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
  1. Epistemic contradiction: $\diamond \alpha \land -\alpha \models \bot$ (if $R$ is state-based)
  2. Non-factivity: $\diamond \alpha \not\models \alpha$
BSML: deontic vs epistemic modals

Proposal

- **Epistemics:** $R$ is state-based
- **Deontics:** $R$ is possibly indisputable (e.g. in performative uses)

Team-sensitive constraints on accessibility relation

- $R$ is **indisputable** in $(M, s)$ iff $\forall w, v \in s : R[w] = R[v]$
  $\iff$ all worlds in $s_M$ access exactly the same set of worlds
- $R$ is **state-based** in $(M, s)$ iff $\forall w \in s : R[w] = s$
  $\iff$ all and only worlds in $s_M$ are accessible within $s_M$
BSML predictions: epistemic and deontic FC

▶ Narrow scope FC: \([\Box (\alpha \lor \beta)]^+ \models \Box \alpha \land \Box \beta\)
▶ Wide-scope FC: \([\Box \alpha \lor \Box \beta]^+ \models \Box \alpha \land \Box \beta\) [if \(R\) is indisputable]

Epistemic modals

▶ \(R\) is state-based, therefore always indisputable:

(33) He might either be in London or in Paris. [+fc, narrow]
(34) He might be in London or he might be in Paris. [+fc, wide]

⇒ narrow and wide scope FC always predicted for pragmatically enriched epistemics

Deontic modals

▶ \(R\) sometimes indisputable, e.g. in performative uses

⇒ narrow scope FC always predicted for enriched deontics
⇒ wide scope FC only predicted if speaker is informed about what is permitted/obligatory [Pesetsky et al. 2017]

Further consequence: all cases of (overt) FC cancellations involve a wide scope configuration in a context where indisputability is not warranted
BSML predictions: overt FC cancellations

- Examples of overt FC cancellations:

  (35) You may eat the cake or the ice-cream, I don’t know which
       You may eat the cake

  (36) You may eat the cake or the ice-cream, it depends on what John
       has taken
       You may eat the cake [Kaufmann 2016]

- Sluicing in (35) and inquisitive *it* in (36) arguably trigger wide scope disjunction in their antecedent [Fusco 2019, Pinton & MA 2022]

  (37) You may eat the cake or the ice-cream, I don’t know which (you
       may eat).

  (38) You may eat the cake or the ice-cream, *it* (= what you may eat)
       depends on what John has taken.

- Sketch of analysis (in BSML + inquisitive disjunction ⨁):

  (a) which/what you may eat ($◊\alpha \lor ◊\beta$) $\rightarrow$ $◊\alpha$
      $◊\beta$

  (b) $◊\alpha \lor ◊\beta \equiv [\exists]$
      $◊\alpha$
      $◊\beta$ $\neq ◊(\alpha \lor \beta)$
Neglect-zero effects in BSML: further predictions

- Modal **D-inferences** are derived: 
  
  - $[\square(\alpha \lor \beta)]^+ \models \Diamond \alpha \land \Diamond \beta$
  
  - But **negative FC** is not predicted:
    
    - $[\neg \square(\alpha \land \beta)]^+ \not\models \Diamond \neg \alpha \land \Diamond \neg \beta$

- In BSML logically equivalent sentences can have different neglect-zero effects, i.e., these effects are **detachable**:

  $$\Diamond (\neg \alpha \lor \neg \beta) \equiv \neg \square (\alpha \land \beta)$$

  $$[\Diamond (\neg \alpha \lor \neg \beta)]^+ \not\equiv [\neg \square (\alpha \land \beta)]^+$$

Only positive disjunction gives rise to FC inference:

$$[\Diamond (\neg \alpha \lor \neg \beta)]^+ \models \Diamond \neg \alpha \land \Diamond \neg \beta$$

$$[\neg \square (\alpha \land \beta)]^+ \not\models \Diamond \neg \alpha \land \Diamond \neg \beta$$
Negative FC (Marty et al., 2021, 2022)

- **Experimental research**: negative FC inferences exist but appear to be less available than positive FC:

  \[(39) \text{Negative FC} \]
  
  a. It is not required that Mia buys both apples and bananas \( \sim \) It is not required that Mia buys apples and that Mia buys bananas
  
  b. \( \neg \Box (\alpha \land \beta) \sim \neg \Box \alpha \land \neg \Box \beta \) \( \equiv \Diamond \neg \alpha \land \Diamond \neg \beta \)

- **BSML\(^+\): BSML + global pragmatic enrichment**

  \[ \alpha \models_{BSML^+} \beta \iff [\alpha]^+ \models_{BSML} [\beta]^+ \]

<table>
<thead>
<tr>
<th>Positive FC</th>
<th>( \Diamond (\alpha \lor \beta) \sim \Diamond \alpha \land \Diamond \beta )</th>
<th>BSML(^+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative FC</td>
<td>( \neg \Box (\alpha \land \beta) \sim \Diamond \neg \alpha \land \Diamond \neg \beta )</td>
<td>weak</td>
</tr>
<tr>
<td>D-inference</td>
<td>( \Box (\alpha \lor \beta) \sim \Diamond \alpha \land \Diamond \beta )</td>
<td>strong</td>
</tr>
<tr>
<td>Negative DI</td>
<td>( \neg \Diamond (\alpha \land \beta) \sim \Diamond \neg \alpha \land \Diamond \neg \beta )</td>
<td>weak</td>
</tr>
<tr>
<td>Low Negative FC</td>
<td>( \Diamond (\neg \alpha \lor \neg \beta) \sim \Diamond \neg \alpha \land \Diamond \neg \beta )</td>
<td>strong</td>
</tr>
</tbody>
</table>

**Table**: Comparison BSML\(^+\) and experimental findings.
Comparison with two recent approaches

- **Goldstein 2019:** FC inferences derived by adding a homogeneity presupposition to the meaning of
  - possibility modal
  - disjunction
    [alternative-based account, Gold19A]
    [dynamic account, Gold19B]
- **Bar-Lev & Fox 2020:** FC inference derived by application of an exhaustivity operator (which includes alternatives on top of negating all the innocently excludable ones)
  [BLF20]

<table>
<thead>
<tr>
<th></th>
<th>BSML⁺</th>
<th>Gold19A</th>
<th>Gold19B</th>
<th>BLF20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive FC</td>
<td>strong</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Negative FC</td>
<td>weak</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Possibility</td>
<td>strong</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Negative Conjunction</td>
<td>weak</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wide Scope FC</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table: Comparison BSML⁺ and alternative approaches

- **BSML⁺ & Gold19B** seem the best options for strong inferences but needs to be supplemented with a theory deriving weak inferences;
- **Within BSML** we can derive both weak and strong inference patterns: BSML⁺ ↦ strong & BSML⁺ * ↦ weak
Modelling neglect-zero effects: different implementations

- More ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function \([\ ]^+\) defined in terms of \(\text{NE} \mapsto \text{BSML}^+\)
  - Model-theoretically, by ruling out \(\emptyset\) from the set of possible states \(\mapsto \text{BSML}^*\)

- Both implementations derive:
  - \(\mapsto\) FC effects (narrow and wide scope FC, the latter with restrictions);
  - \(\mapsto\) cancellations of FC effects under negation (dual prohibition).

- But empirical and conceptual differences:
  - Only BSML* predicts **Negative FC**: \(\neg \Box(\alpha \land \beta) \leadsto \neg \Box \alpha \land \neg \Box \beta\)
  - Only in BSML, where \([\ ]^+\) and \(\emptyset\) are parts of the building blocks, **locality** and **suspension** of neglect-zero effects can be modeled

- Conjecture: neglect-zero can cause two kinds of effects:
  1. weak non-detachable effects (modelled by BSML*);
  2. more robust detachable effects (modelled by \(\text{BSML}^+\)).
The resulting picture

- A pluralism of systems which can be used to model interpretation strategies & reasoning styles people may adopt in different circumstances:
  1. BSML$^\emptyset$: modelling logical-mathematical reasoning where neglect-zero effects are suspended;
  2. BSML$^+$: modelling strong (detachable) neglect-zero effects;
  3. BSML$^*$: modelling weak (global, non-detachable) neglect-zero effects;
  4. . .

- Experimentally testable predictions arising from these conjectures

<table>
<thead>
<tr>
<th></th>
<th>BSML$^\emptyset$</th>
<th>BSML$^+$</th>
<th>BSML$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS FC</td>
<td>$\Diamond(\alpha \lor \beta) \leadsto \Diamond \alpha \land \Diamond \beta$</td>
<td>s</td>
<td>-</td>
</tr>
<tr>
<td>Dual prohibition</td>
<td>$\neg \Diamond(\alpha \lor \beta) \leadsto \neg \Diamond \alpha \land \neg \Diamond \beta$</td>
<td>s</td>
<td>+</td>
</tr>
<tr>
<td>Negative FC</td>
<td>$\neg \Box(\alpha \land \beta) \leadsto \neg \Box \alpha \land \neg \Box \beta$</td>
<td>w</td>
<td>-</td>
</tr>
<tr>
<td>WS FC</td>
<td>$\Diamond \alpha \lor \Diamond \beta \leadsto \Diamond \alpha \land \Diamond \beta$</td>
<td>?</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table:** Comparison BSML$^\emptyset$, BSML$^+$ and BSML$^*$. 
Conclusions

- **Free choice**: a mismatch between logic and language
- **Grice’s insight**: stronger meanings can be derived paying more “attention to the nature and importance to the conditions governing conversation”
- **Standard implementation**: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning
  
  Elegant picture, but, when applied to $\text{FC} \& \text{ignorance inferences}$, empirically inadequate
- **My proposal**: $\text{FC} \& \text{related inferences as neglect-zero effects}$
  
  Literal meanings (NE-free fragment) + pragmatic factors (NE) $\Rightarrow \text{FC} \& \text{possibility}$
  
  Implementation in BSML (a team-based modal logic)
  A pluralism of systems representing different reasoning styles:
  
  $\text{BSML}^* \text{ vs BSML}^+ \text{ vs BSML}^\emptyset$
Collaborators & related (future) research

**Logic**

Proof theory (Anttila, Yang, Knudstorp); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Bentham, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); Aristotelian logic in qBSML\(\rightarrow\) (MA);...

**Language**

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); ...

**Thank You!**

---

4This work was supported by (i) Nothing is Logical (NihiL), an NWO OC project (grant no 406.21.CTW.023) and (ii) PLEXUS, (Grant Agreement no 101086295) a Marie Sklodowska-Curie action funded by the EU under the Horizon Europe Research and Innovation Programme.