N∅thing is Logical

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N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of enrichments albeit not (always) of the canonical Gricean kind

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KONINKLIJKE NEDERLANDSE AKADEMIE VAN WETENSCHAPPEN

PLEXUS
PHILOSOPHICAL, LOGICAL, AND EXPERIMENTAL ROUTES TO SUBSTRUCTURALITY

NWO

Non-classical inferences

Free choice (FC)

(1) FC: $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [von Wright 1968]

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is either in the garden *or* in the attic $\rightsquigarrow$ The prize might be in the garden *and* might be in the attic [Grice 1989, p.45]

(5) ? I have two *or* three children.

- In the standard approach, ignorance is a conversational implicature
- Less consensus on FC inferences analysed as conversational implicatures; grammatical (scalar) implicatures; semantic entailments; ...

The challenge of FC: adding FC to classical modal logic implies the equivalence of any two possibility claims

$$\Diamond a \ \Rightarrow_{\text{CML}} \ \Diamond(a \vee b) \ \Rightarrow_{\text{FC}} \ \Diamond b$$

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ grammatical (scalar) implicatures]

- They are rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers construct models depicting reality (some verifying the sentence, some falsifying it) $\mapsto$ common assumption
  and in this process tend to neglect models that verify the sentence by virtue of an empty configuration (*zero-models*) $\mapsto$ novel hypothesis

- Tendency to neglect zero-models follows from the cognitive difficulty of:

  1. conceiving emptiness, the absence of things rather than their presence
  2. evaluating truths with respect to empty witness sets

[Nieder 2016; Bott *et al* 2019]

Novel hypothesis: neglect-zero

Illustration

(6) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [□, □, □]; [■, ■, ■]; [△, △, △]; [▲, ▲, ▲]; ...

Zero-models in (6-c) verify the sentence by virtue of an empty set of black squares

- Cognitive difficulty of zero-models confirmed by experimental findings and connected to / can be argued to explain:
  - the special status of 0 among the natural numbers [Nieder 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott *et al* 2019]
- NZ hypothesis: neglect-zero also at the origin of many common departures from classical reasoning
  - FC and ignorance [MA 2022]
  - Existential Import: every A is B $\Rightarrow$ some A is B
  - Aristotle's Thesis: if not A then A $\Rightarrow \perp$
  - Boethius' Thesis: if A then B & if A then not B $\Rightarrow \perp$

[Ziegler, Knudstorp & MA 2025]

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(7) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(8) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(9) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(10) M ate an apple or a banana.
- a. Verifier: ?
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: ?

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(11) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(12) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(13) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(14) M ate an apple or a banana.
- a. Verifier: ?
- b. Falsifiers: [🍐]; [ ]
- c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (14-c): verify the sentence by virtue of an empty witness for one of the disjuncts

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(15) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(16) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(17) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(18) M ate an apple or a banana.
- a. Verifier: [🍎 | 🍌] $\Leftarrow$ 'split'
- b. Falsifiers: [🍐]; [ ]
- c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (18-c): verify the sentence by virtue of an empty witness for one of the disjuncts
- **Split state** in (18-a): simultaneously entertains different (possibly conflicting) alternatives
- Neglect-zero hypothesis: ignorance and FC arise because split states emerge as natural verifiers for disjunctions since zero-models, where only one of the disjuncts is depicted, are cognitively taxing and therefore kept out of consideration

A new conjecture: no-split

(19) Maria ate an apple or a banana.

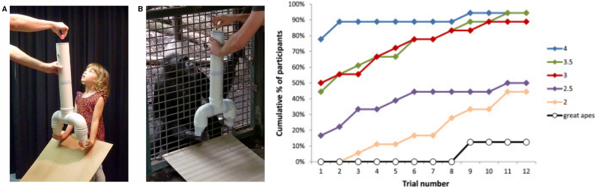
a. Verifier: [🍎 | 🍌] [⇐ **split state**]

b. Falsifiers: [🍐]; [ ]

c. **Zero-models**: [🍎]; [🍌]

- **Split states**: multiple alternative possibilities processed in a parallel fashion $\mapsto$ also a cognitively taxing operation

No-split conjecture [Klochowicz, Sbardolini & MA, SuB 2025]
the ability to split states (entertain multiple possibilities) is developed late



Children have trouble conceiving multiple possibilities [Redshaw & Suddendorf 2016]

- Combination of neglect-zero + no-split can explain non-classical inferences observed in pre-school children

A new conjecture: no-split

- Pre-school children sometimes (but systematically) interpret disjunctions conjunctively [Singh *et al* 16 (but cf Skordos *et al* 20); Cochard 25; Bleotu *et al* 25]

(20) M ate an apple or a banana = M ate an apple and a banana
$(\alpha \vee \beta) \equiv (\alpha \wedge \beta)$

(21) M can eat an apple or a banana = M can eat an apple and a banana
$\Diamond(\alpha \vee \beta) \equiv \Diamond(\alpha \wedge \beta) \not\equiv \Diamond\alpha \wedge \Diamond\beta$

(22) M didn't eat an apple or/and a banana = M neither ate an apple nor a banana
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \equiv \neg(\alpha \wedge \beta)$

- Proposal: children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, do not have the ability to split

1. Deriving ignorance:

   Apple OR banana $\Rightarrow_{\text{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\text{SPLIT}}$ [🍎 | 🍌]

   $\rightsquigarrow$ It might be an apple and it might be a banana (adults)

2. Deriving conjunctive reading:

   Apple OR banana $\Rightarrow_{\text{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\text{NO-SPLIT}}$ [🍎 🍌]

   $\rightsquigarrow$ Both an apple and a banana (children)

3. In case of incompatible alternatives: [Leahy & Carey 2020]

   Left OR right $\Rightarrow_{\text{NZ}}$ ↙ + ↘ $\Rightarrow_{\text{NO-SPLIT}}$ **contradiction** ($\perp$)

   $\rightsquigarrow$ Random singular guess (children)

Cognitive bias approach

Common assumption: Reasoning and understanding of natural language involve the creation of mental models [e.g., Johnson-Laird 1983]

- **Understanding** a sentence S means being able to mentally construct a model picturing the world which verifies S, and possibly also a model which falsifies it
- **Reasoning** depends on two main processes: first construct verifying models for the premises and then check the validity of the conclusion on these models

Novel hypothesis: biases can constrain the construction of these models and therefore impact both reasoning and interpretation:

- **Neglect-zero** prevents the constructions of zero-models;
- **No-split** expresses a dispreference for split-states.

Comparison with competing accounts

	Ignorance	**FC & DIST**	**ES-Quant**	**Scalar impl.**	**Conjunctive *or***
Neo-Gricean	reasoning	reasoning	reasoning	reasoning	—
Grammatical	debated	grammatical	grammatical	grammatical	grammatical
Cognitive bias	neglect-zero	neglect-zero	neglect-zero	—	negl-z + no-split

Next

- Logical modelling of biases in team semantics
- Experimental findings
  - Degano *et al* (Nat Lang Sem, 2025): ignorance $\Leftarrow$
  - Klochowicz *et al* (CogSci25, SuB25): on scalar, DIST & ES-Quant
  - Bleotu *et al* (TbiLLC 2025): on conjunctive *or*

Modelling biases in team semantics

General methodology

Natural language sentences translated into classical logic formulas interpreted in a team semantics which models both classical and enriched interpretations

$$\textsc{Natural Language} \quad \mapsto \quad \textsc{Logical Language} \quad \begin{array}{l} \nearrow \quad \alpha \quad \textbf{(classical)} \\ \searrow \quad [\alpha]^{+} \ \textbf{(enriched)} \end{array}$$

Back to FC

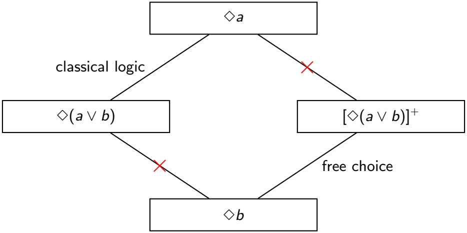


Figure: FC derived only for NZ enriched formulas

Modelling biases in team semantics

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]
  - Classical modal logic: $[M = \langle W, R, V \rangle]$

$$M, w \models \phi, \text{ where } w \in W$$

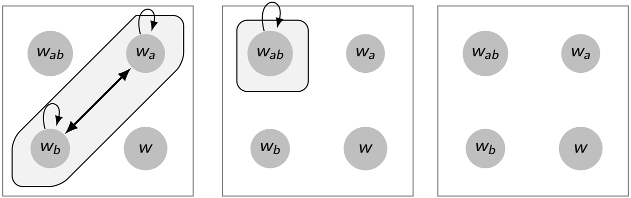
  - Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

- Two crucial features
  - The empty set is among the possible teams ($\emptyset \subseteq W$) $\mapsto$ zero-models
  - Multi-membered teams can model parallel processing of alternatives $\mapsto$ split states

Illustrations

$$[A = \{a, b\}; W = \{w_{ab}, w_a, w_b, w\}]$$



(a) split (b) no-split (c) empty team

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Modelling neglect-zero & no-split

- Model-theoretically:
  - by disallowing empty (neglect-zero) and multi-membered teams (no-split)
- Syntactically: via new logical atoms/operators
  - Neglect-zero: via **non-emptiness atom** NE which disallows empty teams as possible verifiers [Yang & Väänänen 2017]

$$M, t \models \mathrm{NE} \text{ iff } t \neq \emptyset$$

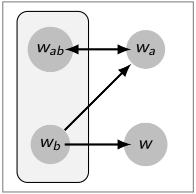
  - No-split: via **flattening operator** F which induces pointwise evaluations and therefore avoids simultaneous processing of alternatives

$$M, t \models \mathrm{F}\phi \text{ iff for all } w \in t : M, \{w\} \models \phi$$

BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$



Bilateral team semantics

Given Kripke model $M = \langle W, R, V\rangle$ & teams/states $s, t, t' \subseteq W$

$$
\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w,p) = 1 \\
M, s \dashv p & \text{iff} & \text{for all } w \in s : V(w,p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi \quad \Leftarrow \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \;\&\; M, s \dashv \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \;\&\; M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \dashv \phi \;\&\; M, t' \dashv \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \dashv \phi \qquad [\text{where } R[w] = \{v \in W \mid wRv\}] \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

Proof Theory: MA, Anttila & Yang (2024); Expressive completeness: Anttila & Knudstorp (2025); Comparisons via translation into Modal Information Logic: Knudstorp *et al* (2025)

Neglect-zero effects in BSML

BSML models both classical and enriched interpretations

- α ($\textsc{ne}$-free) $\Rightarrow$ empty team allowed $\mapsto$ **classical**
- $[\alpha]^+$ $\Rightarrow$ empty team not allowed $\mapsto$ **enriched**

Neglect-Zero enrichment function

For $\textsc{ne}$-free α, $[\alpha]^+$ defined as follows:

$$
\begin{array}{rcl}
[p]^+ & = & p \wedge \textsc{ne} \\
[\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \textsc{ne} \\
[\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \textsc{ne} \\
[\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \textsc{ne} \\
[\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \textsc{ne}
\end{array}
$$

$[\ ]^+$ enriches formulas with the requirement to satisfy $\textsc{ne}$ (non-emptiness) distributed along each of their subformulas

Formal characterization of neglect-zero effects

$\alpha \rightsquigarrow_{nz} \beta$ (β is a neglect-zero effect of α) iff $\alpha \not\models \beta$ but $[\alpha]^+ \models \beta$

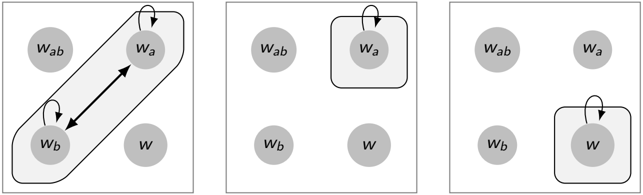
Formal characterization of zero and no-zero models

(M, s) is a zero-model for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$
(M, s) is a no-zero verifier for α iff $M, s \models [\alpha]^+$

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi$$



Verifier
$\models a \vee b$; $\models [a \vee b]^+$

Zero-model
$\models a \vee b$; $\not\models [a \vee b]^+$

Falsifier
$=\!\!| \; a \vee b$; $=\!\!| \; [a \vee b]^+$

Figure: Models for $a \vee b$

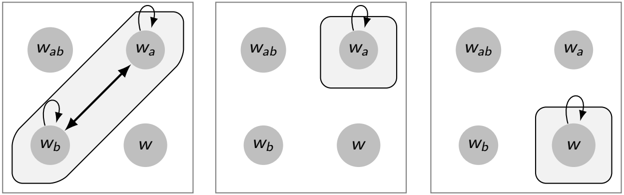
Why is $\{w_a\}$ a zero-model?

- Empty team allowed $\mapsto$ substates can be empty **(classical)**
  $\{w_a\} \models a \vee b$ by virtue of an empty witness for b, $M, \emptyset \models b$
- Empty team not allowed $\mapsto$ substates cannot be empty **(enriched)**
  $\{w_a\} \not\models [a \vee b]^+$ because there is no non-empty subset supporting b

Neglect-zero effects in BSML: enriched disjunction

- s supports an **enriched disjunction** $[\alpha \vee \beta]^+$ iff s is the union of two non-empty substates, each supporting one of the disjuncts

$$[\alpha \vee \beta]^+ = (\alpha \wedge \text{NE}) \vee (\beta \wedge \text{NE}) \wedge \text{NE}$$



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\dashv [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities [Zimmermann 2000]

(23) M ate an apple or a banana $\rightsquigarrow_{nz}$ It might be an apple and it might be a banana

$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where R is state-based)

- Main result: in BSML [ ]$^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions [MA 2022]
  - $\mapsto$ we derive ignorance, FC and related effects (for enriched formulas);
  - $\mapsto$ [ ]$^+$-enrichment vacuous under single negation.

Neglect-zero effects in BSML: main results

After enrichment

- We derive ignorance, FC and related inferences:
  - Ignorance: $[\alpha \vee \beta]^+ \models \Diamond_e\alpha \wedge \Diamond_e\beta$ (if R is state-based)
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Double prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007]
  - Epistemic contradiction: $\Diamond_e\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  - Non-factivity: $\Diamond_e\alpha \not\models \alpha$

Team-based constraints on accessibility relation

- R state-based in (M, s) iff all and only worlds in s are accessible within s [$\mapsto$ **epistemics** (always)]
- R indisputable in (M, s) iff all worlds in s access exactly the same set of worlds [$\mapsto$ **deontics** (sometimes)]

The data

(24) Double Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream $\rightsquigarrow$ You are not allowed to eat either one

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(25) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus $\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \to \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \to (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(26) Wide Scope FC [Zimmermann 2000, Cremers *et al* 2017]

a. Detectives may go by bus or they may go by boat $\rightsquigarrow$ Detectives may go by bus and may go by boat

b. Mr. X might be in Victoria or he might be in Brixton $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (if R indisputable)

(27) FC cancellation [sluice indicates wide scope disjunction]

a. Detectives may go by bus or by boat, I don't know which $\not\rightsquigarrow$ Detectives may go by bus and may go by boat

b. $\Diamond\alpha \vee \Diamond\beta \not\rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (if R not indisputable)

Experimental findings

Cognitive bias view

Non-classical inferences prominently explained by neo-Gricean or grammatical mechanisms are instead consequence of a neglect-zero (+ no-split) tendency

Comparison with competing accounts[1]

	Ignorance	**FC & DIST**	**ES-Quant**	**Scalar impl.**	**Conjunctive *or***
Neo-Gricean	reasoning	reasoning	reasoning	reasoning	—
Grammatical	debated	grammatical	grammatical	grammatical	grammatical
Cognitive bias	neglect-zero	neglect-zero	neglect-zero	—	negl-z + no-split

Recent experiments

1. Degano *et al* (Nat Lang Sem, 2025): ignorance ⇐
2. Klochowicz *et al* (CogSci25, SuB25): on DIST, ES-Quant & scalar

   (28) a. Each square is red or white ⇒ there are white squares and red squares [DIST]
   b. Less than 3 squares are black ⇒ there are some black squares [ES-Quant]
   c. Some of the squares are black ⇒ not all of the squares are black [scalar]

   Main result:
   - Semantic priming between DIST and ES-Quant;
   - No priming between scalar and ES-Quant.
3. Bleotu *et al* (TbiLLC 2025): on conjunctive *or*

[1] Neo-Gricean: Horn, Soames, Sauerland, ... Grammatical view: Chierchia, Fox, Singh *et al*, ...

Back to plain disjunction

Enriched meanings for disjunction

(29) Maria ate an apple or a banana $\rightsquigarrow$ $(\alpha \vee \beta)$

a. Scalar implicature: not both $\neg(\alpha \wedge \beta)$

b. Conjunctive interpretation: both $(\alpha \wedge \beta)$

c. Ignorance: speaker doesn't know which ?

Two components of full ignorance: possibility vs uncertainty

(30) Maria ate an apple or a banana $\rightsquigarrow$ speaker doesn't know which [Degano *et al* 2025][2]

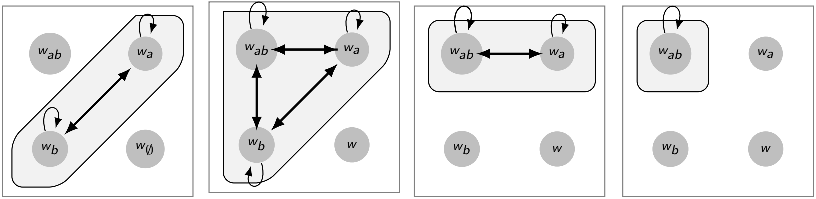
a. Possibility: It is possible that M ate an apple and it is possible that M ate a banana $\Diamond_e \alpha \wedge \Diamond_e \beta$

b. Uncertainty: It is uncertain that M ate an apple and it is uncertain that M ate a banana $\neg\Box_e \alpha \wedge \neg\Box_e \beta$

[2] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. "The ups and downs of ignorance." *Natural Language Semantics*, 2025.

Neglect-zero effects on disjunction: predictions of BSML

Many no-zero verifiers for enriched disjunction



(d) no-zero & scalar $\models \neg(a \wedge b)$ (e) no-z & non-scalar $\not\models \neg(a \wedge b)$ (f) no-z & no-uncertain $\not\models \neg\Box_e a$ (g) no-zero & no-split $\models (a \wedge b)$

Figure: Models for enriched $[a \vee b]^+$.

1. Neglect-zero enrichment derives **possibility**: $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$
2. Neglect-zero enrichment does not derive scalar implicatures;
3. Neglect-zero enrichment does not derives uncertain inferences $\mapsto$ in contrast to standard neo-Gricean approach to ignorance $\Leftarrow$
4. **No-split** verifiers compatible with neglect-zero enrichments
   - **No-split** conjecture: only **no-split** verifiers accessible to 'conjunctive' pre-school children [Klochowicz, Sbardolini, MA, SuB, 2025]

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

(i) Uncertainty derived through quantity reasoning

(31) $\alpha \vee \beta$ ASSERTION

(32) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

(ii) Possibility derived from uncertainty and quality about assertion

(33) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

(34) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Neglect-zero derivation

(i) Possibility derived as neglect-zero effect

(35) $\alpha \vee \beta$ ASSERTION

(36) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

(ii) Uncertainty derived from possibility and scalar reasoning

(37) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

(38) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

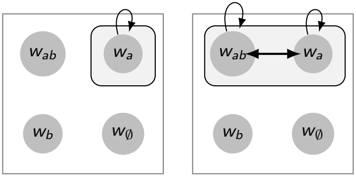
Neo-Gricean vs neglect-zero explanation

Contrasting predictions of competing accounts of ignorance

- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano *et al* 2025]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs **44%**)
  - ⇒ Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-Gricean approach



(a) **zero-model** (b) no-uncertain

Figure: Models for $(a \vee b)$

Conclusions

- FC, ignorance: a mismatch between logic and language
- Grice's insight:
    - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: some non-classical inferences due to cognitive bias rather than Gricean reasoning
    - FC, possibility and related inferences as neglect-zero effects

      Literal meanings (classical fragment) + cognitive factor (NE) $\Rightarrow$ FC, possibility, etc

    - Conjunctive *or* as no-zero + no-split effect

      Literal meanings (classical fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in (extensions of) BSML, a team-based modal logic
- Recent experiments provide some first tentative evidence in agreement with the neglect-zero hypothesis
- Appendix:
    - Experimenting with disjunction and quantifiers
    - Comparison via translation into Modal Information Logic

Collaborators & related (future) research


Anttila


Degano


Klochowicz


Knudstorp


Ramotowska


Zhou

& many more . . .

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); dynamics (MA); typed BSML (Muskens); connexive logic (Knudstorp, Ziegler & MA); belief revision (Klochowicz)

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs, Ziegler); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); acquisition (Klochowicz, Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo, Spychalska, Szymanik, Visser); . . .

THANK YOU![3]

[3] This work is supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

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Neglect-zero effects on quantifiers: Empty Set (ES) inferences

Predictions of qBSML$^{\to}$[4]

(39) Less than three squares are black $\mapsto \forall xyz((Sx \wedge Bx \wedge \dots) \to (x = y \vee \dots))$

a. Verifier: [■, □, ■]

b. Falsifier: [■, ■, ■]

c. Zero-models: [□, □, □]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are black squares

(40) Every square is black. $\mapsto \forall x(Sx \to Bx)$

a. Verifier: [■, ■, ■]

b. Falsifier: [■, □, ■]

c. Zero-models: [△, △, △]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are squares

(41) No squares are black. $\mapsto$ (i) $\forall x(Sx \to \neg Bx)$; (ii) $\neg \exists x(Sx \wedge Bx)$

a. Verifier: [□, □, □]

b. Falsifier: [■, □, □]

c. Zero-models for (i): [△, △, △]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are squares

d. Zero-models for (ii): none no neglect-zero effect

(42) Every square is red or white. $\mapsto \forall x(Sx \to (Rx \vee Wx))$

a. Verifier: [■ (red), □, ■ (red)]

b. Falsifier: [■ (red), □, ■]

c. Zero-models: [■ (red), ■ (red), ■ (red)]; [□, □, □]; … $\rightsquigarrow_{nz}$ there are white & red squares

These predictions tested in Bott, Klochowicz, Schlotterbeck *et al* (2024, 2025)

[4] MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023).

Experimenting with quantifiers and disjunction

Four non-classical interpretations

(43) a. Some of the squares are black ⇒ not all of the squares are black [scalar UB]
b. Each square is red or white ⇒ there are white squares and red squares [DIST]
c. Less than 3 squares are black ⇒ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black ⇒ there are some squares [ES-restrictor]

Three competing accounts

	UB	DIST	ES-scope	ES-restrictor
Alternative-based	implicature	implicature	implicature	implicature
Bott *et al*, 2019	—	—	neglect-zero	presupposition
Nihil	—	neglect-zero	neglect-zero	neglect-zero

Two experiments

- Exp 1: Answering questions about the emptyset (O. Bott *et al*, SuB 2024)
- Exp 2: Priming with zero-models (Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025)

Three main conclusions

1. Evidence that ES-restrictor is a presupposition (Exp 1)
2. Evidence that UB differs from both ES-scope and DIST (Exp1 and Exp2)
3. Some evidence that ES-scope and DIST involve the same cognitive process (Exp 2)

Experimenting with quantifiers and disjunction

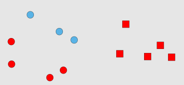
Non-classical interpretations

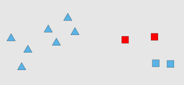
(44)
a. Some of the squares are black $\Rightarrow$ not all of the squares are black [UB]
b. Each square is red or white $\Rightarrow$ there are white squares and red squares [DIST]
c. Less than 3 squares are black $\Rightarrow$ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black $\Rightarrow$ there are some squares [ES-restrictor]

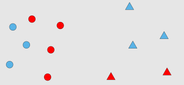
Exp1: Bott et al, SuB 2024

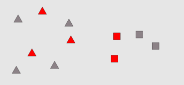
- Question-answer task:

(45) Ist jedes Dreieck entweder rot oder blau? Ja/Nein/Komische Frage
(Is every triangle either red or blue?) Yes/No/Odd question


empty restr


DIST target (zero-model)


control '*yes*'


control '*no*'

- Main results:
  1. Evidence that ES-restrictor is a presupposition: questions in empty restrictor models uniformly perceived as odd
  2. ES-scope (37%) and DIST (23%) unaffected by question environment; UB much less available (10%, while 40% when unembedded)
  3. Inconclusive evidence on whether ES-scope and DIST had the same source

Experimenting with quantifiers and disjunction

Non-classical interpretations

(46) a. Some of the squares are black $\Rightarrow$ not all of the squares are black [UB/scalar]
b. Each square is red or white $\Rightarrow$ there are white and red squares [DIST]
c. At most 2 squares are black $\Rightarrow$ there are some black squares [ES-scope, sup]
d. Less than 3 squares are black $\Rightarrow$ there are some black squares [ES-scope, comp]

Two competing accounts

	UB	DIST	ES-scope
Alternative-based	implicature	implicature	implicature
Nihil ($\text{qBSML}^{\rightarrow}$)	—	neglect-zero	neglect-zero

Exp2: Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025

- Tested whether frequency of enrichment in (46-d) changed after participants were primed to suspend other enrichments in (46-a-c):
  - UB $\Rightarrow$ ES-scope[c]; DIST $\Rightarrow$ ES-scope[c]; ES-scope[s] $\Rightarrow$ ES-scope[c]
- Results:
  1. Semantic priming between DIST and ES-scope (comp)
  2. No priming between UB and ES-scope (comp)
  3. No trial-to-trial priming from ES-scope (sup) to ES-scope (comp) but spill-over and adaptation effects
- Tentative conclusion: ES-scope and DIST (but not UB) involve the same cognitive process, as predicted by neglect-zero hypothesis

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \;\&\; M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & possibility semantics: description of ontological structures in the world
  - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- Next:
  - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[5] common ground where related systems can be interpreted and their connections and differences can be explored
- Goal: translations into (extensions of) MIL of the following systems:
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - BSML
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Here focus on propositional fragments
  - disjunction
  - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[5] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

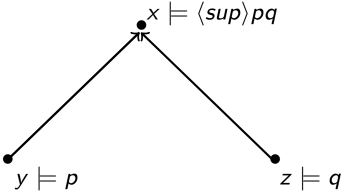
$$\begin{array}{rcl}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ and } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ or } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \;\&\; \mathcal{M}, y \models \phi \;\&\; \mathcal{M}, z \models \psi
\end{array}$$

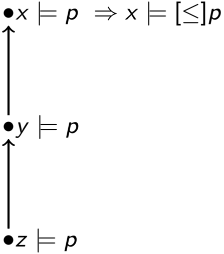
$$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \;\Rightarrow\; \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)[6]

$$
\begin{array}{rcl}
& \vdots & \\
tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\
tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\
tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\
& \vdots &
\end{array}
$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$
\begin{array}{rcl}
& \vdots & \\
tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\
tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\
tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\
& \vdots &
\end{array}
$$

[6] Johan van Benthem, Nick Bezhanishvili, Wesley H. Holliday, A bimodal perspective on possibility semantics, *Journal of Logic and Computation*, Volume 27, Issue 5, July 2017, Pages 1353–1389.

Translations into Modal Information Logic

- Truthmaker semantics (Fine)[7]

$$
\begin{array}{rcl}
 & \dots & \\
(\neg\phi)^+ & = & (\phi)^- \\
(\neg\phi)^- & = & (\phi)^+ \\
(\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\
(\phi \vee \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\
(\phi \wedge \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^-
\end{array}
$$

- BSML

$$
\begin{array}{rcl}
 & \dots & \\
(\neg\phi)^+ & = & (\phi)^- \\
(\neg\phi)^- & = & (\phi)^+ \\
(\phi \vee \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\
(\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\
(\phi \wedge \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\
 & \dots &
\end{array}
$$

[7] van Benthem, Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic* (2019).

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/tensor/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics, operational semantics for Positive R]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic/inquisitive logic[8]
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)
    - Cofinal disjunction and intuitionistic negation (possibility semantics)

[8] Inquisitive & intuitionistic logic: same connectives but different translations for the atoms.