Neglect-zero effects at the semantics-pragmatics interface

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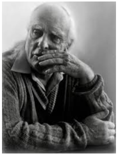
Logic and Conversation

(1) ALAN: Are you going to Paul's party? [Davis, 2019]
BARB: I have to work.
a. $\rightsquigarrow$ Barb has to do something [$\Rightarrow$ semantics]
b. $\rightsquigarrow$ Barb is not going to Paul's party [$\Rightarrow$ pragmatics]

Grice's paradise

Canonical divide between semantics and pragmatics

- **Pragmatic inference**: cancellable, non-embeddable, non-detachable, ... [$\Rightarrow$ derivable by conversational factors]
- **Semantic inference**: non-cancellable, embeddable, detachable, ... [$\Rightarrow$ derivable by classical logic]


H. P. Grice (1913–1988)

Beyond the canonical divide

- Gricean picture recently challenged by a class of modal inferences triggered by existential/disjunctive constructions:
  - Ignorance inference in modified numerals and epistemic indefinites;

(2) a. Aicha has *at least two* degrees $\rightsquigarrow$ speaker does't know how many [Geurts & Nouwen 2007]

b. ?I have *at least two* children.

(3) *Irgendjemand* hat angerufen. #Rat mal wer?
Irgend-someone has called Guess prt who
Someone called $\rightsquigarrow$ speaker doesn't know who [Haspelmath 1997]

  - Free choice inferences in disjunction and indefinites;

(4) You may go to the beach *or* to the cinema $\rightsquigarrow$ you may go to the beach *and* you may go to the cinema. [Kamp 1973]

(5) Maria muss *irgendeinen* Arzt heiraten.
Maria must irgend-one doctor marry
Mary must marry a doctor $\rightsquigarrow$ any doctor is permissible
[Kratzer & Shimoyama 2002]

- Common core of these inferences:
  - Although derivable by conversational factors they lack other defining properties of pragmatic inferences [$\mapsto$ **inferences of the 3rd kind**]

Beyond Gricean paradise

		pragm. derivable	**cancel lable**	**non-embed.**	**proc. cost**	**acqui sition**
Pra gma tics	Conversational implicature B has to work $\rightsquigarrow$ B is not coming to party	+	+	+	high	late
Sem ant ics	Classical entailment I read some novels $\rightsquigarrow$ I read something	−	−	−	low	early
3rd Kind	Epistemic indefinites *Irgendjemand* hat angerufen $\rightsquigarrow$ Speaker doesn't know who	+	−	+	?	?
	FC disjunction You may do A *or* B $\rightsquigarrow$ You may do A	+	?	?	low	early
	Scalar implicature I read *some* novels $\rightsquigarrow$ I didn't read all novels	+	+	?	high	late

N∅thing is Logical (NihiL)

- Goal of the project: a formal account of 3rd kind inferences which captures their quasi-semantic behaviour while explaining their pragmatic nature
- Strategy: develop **logics of conversation** which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic factor

> *One final remark: my specific motivation for developing this account of indicative conditionals is of course to solve a puzzle [. . .] But I have a broader motivation which is perhaps more important. That is to defend, by example, the claim that the concepts of pragmatics (the study of linguistic contexts) can be made as mathematically precise as any of the concepts of syntax and formal semantics; to show that one can recognize and incorporate into abstract theory the extreme context dependence which is obviously present in natural language without any sacrifice to standards of rigor [Stalnaker, 1975, Indicative Conditionals]*

Novel hypothesis: neglect-zero

- FC and ignorance inferences are
  - neither the result of conversational reasoning (as proposed in neo-gricean approaches) [$\neq$ canonical conversational implicatures]
  - nor the effect of optional applications of grammatical operators (as in the grammatical view) [$\neq$ scalar implicatures]
- Rather they are a straightforward consequence of something else speakers do in conversation, namely,
  - when interpreting a sentence they create structures representing reality, and in doing so they systematically neglect structures which verify the sentence by virtue of some empty configuration (*zero-models*)
- This tendency, which I call **neglect-zero**, follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets (Nieder 2016, Bott et al, 2019)

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

- Cognitive difficulty of zero-models confirmed by findings from number cognition and also explains
  - the special status of 0 among the natural numbers (Nieder, 2016)
  - existential import effects operative in the logic of Aristotle (*every square is black* ⇒ *some square is black*) (Geurts, 2007)
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (Bott et al., 2019)
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and ignorance inferences

Novel hypothesis: neglect-zero

Comparison with competing accounts

	Ignorance inference	**FC inference**	**Scalar implicature**
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
MA proposal	neglect-zero	neglect-zero	—

Arguments in favor of neglect-zero hypothesis

- Cognitive plausibility: differences between FC and scalar implicatures (Chemla & Bott, 2014; Tieu et al, 2016):

	processing cost	**acquisition**
FC inference	low	early
scalar implicature	high	late

  - Expected on neglect-zero hypothesis:
    - FC inference follows from the assumption that when interpreting sentences language users neglect zero-models
    - Zero-models neglected because cognitively taxing
  - Harder to explain on neo-Gricean or grammatical view
- Empirical coverage: Dual prohibition, Universal FC, Double negation FC, Wide scope FC, Modal disjunction (ignorance) and Negative FC.

The paradox of free choice

- Free choice permission in natural language:

(8) You may (A or B) $\rightsquigarrow$ You may A

- But (9) not valid in standard deontic logic (von Wright 1968):

(9) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(10)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (10) uses the following valid principle:

(11) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$

- Natural language counterpart of (11), however, seems invalid:

(12) You may post this letter $\not\rightsquigarrow$ You may post this letter or burn it. [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice Permission:

(13)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by addition + monotonicity]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic solutions [$\Rightarrow$ keep logic as is]
  - FC inferences are pragmatic inferences (conversational implicatures)
  - $\Rightarrow$ step leading to 3 is unjustified
- Grammatical solutions [$\Rightarrow$ keep logic as is]
  - FC inferences result from application of covert grammatical operator
  - $\Rightarrow$ step leading to 3 is unjustified
- Semantic solutions [$\Rightarrow$ change the logic]
  - FC inferences are semantic entailments
  - $\Rightarrow$ step leading to 3 is justified, but step leading to 2 is no longer valid (or transitivity fails)
- Neglect-zero [$\Rightarrow$ change the logic]
  - FC inferences as neglect-zero effects

(14)
1. $\Diamond a$
2. $\Diamond(a \vee b) \quad \neq \quad \Diamond(a \vee b)^{+neglect\text{-}zero}$
3. $\qquad\qquad\qquad \Diamond b$

Free choice: syntax, semantics or pragmatics

Free choice effects systematically disappear in negative contexts:

(15) Dual Prohibition (Alonso-Ovalle 2005)

a. You are not allowed to eat the cake or the ice-cream
$\rightsquigarrow$ You are not allowed to eat either one

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

- Unexpected on a semantic account like Aloni (2007) where $\Diamond(\alpha \vee \beta) \models \Diamond\alpha \wedge \Diamond\beta$
- Predicted by pragmatic accounts: pragmatic inferences do not embed under logical operators
- Derivable by grammatical approaches by assuming additional constraints (e.g., Strongest Meaning Hypothesis)

Free choice: syntax, semantics or pragmatics

Free choice effects embeddable under universal quantification:

(16) Universal FC (Chemla 2009)

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

- Unexpected on a pragmatic account: pragmatic inferences do not embed under logical operators
- Predicted by semantic accounts where $\Diamond(\alpha \vee \beta) \models \Diamond\alpha \wedge \Diamond\beta$
- Unproblematic for grammatical accounts

Free choice: syntax, semantics or pragmatics

Free choice under non-monotonic operators:

(17) All-others FC (Gotzner et al. 2020)

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

- Predicted by accounts which validate
  - Dual prohibition: $\neg\Diamond(\alpha \vee \beta) \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$
  - Double negation FC: $\neg\neg\Diamond(\alpha \vee \beta) \models \Diamond\alpha \wedge \Diamond\beta$
- Problematic for many accounts (e.g., Aloni, 2007), including grammatical ones (Bar-Lev & Fox 2020 fails to capture the case of *exactly two*, Gotzner et al., 2020)

Free choice: syntax, semantics or pragmatics

Free choice effects also arise with wide scope disjunctions:

(18) Wide Scope FC (Zimmermann 2000)

a. Detectives may go by bus or they may go by boat. $\rightsquigarrow$ Detectives may go by bus and may go by boat.
b. Mr. X might be in Victoria or he might be in Brixton. $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.
c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

- Wide scope FC hard to capture (not derivable by plain Gricean reasoning)
- Standard strategy: wide scope FC reduced to narrow scope FC:

(19)
a. Detectives may go by bus or they may go by boat.
b. Logical Form: $\Diamond(\alpha \vee \beta)/\#\Diamond\alpha \vee \Diamond\beta$

- But then (20) would require dubious syntactic operations:

(20)
a. You may email us or you can reach the Business License office at 949 644-3141. $\rightsquigarrow$ You may email us
b. Logical Form: $\Diamond\alpha \vee \Diamond\beta/\#\Diamond(\alpha \vee \beta)$

(Simons' covert ATB movement would not work here, Alonso-Ovalle 2006)

Free choice: summary data and predictions

(21)

a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]

c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]

d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]

e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Semantic	yes	no	yes	no	no
Pragmatic	yes	yes	no	?	no
Grammatical	yes	yes	yes	no	no

Free choice: syntax, semantics or pragmatics

- My proposal: a logic-based approach
  - FC inference derived by modelling the intrusion of neglect-zero [tendency to neglect zero-models] in the process of interpretation
- Neglect-zero: cognitive factor operative in conversation
- Neglect-zero effects modeled using tools from team semantics

Teams

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Väänänen 2007; Yang & Väänänen 2017]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states
- Assertion & rejection conditions are modeled rather than truth

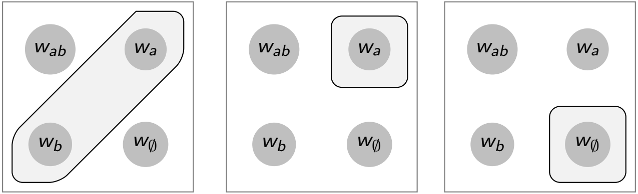
$$M, s \models \phi, \text{ ``}\phi \text{ is assertable in } s\text{''}, \quad \text{with } s \subseteq W$$
$$M, s \dashv \phi, \text{ ``}\phi \text{ is rejectable in } s\text{''}, \quad \text{with } s \subseteq W$$

- Inferences relate speech acts rather than propositions and therefore might diverge from semantic entailments

Neglect-zero effects in BSML: core idea

- A state s supports a **disjunction** $\phi \vee \psi$ iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \ \& \ M, t \models \phi \ \& \ M, t' \models \psi$$



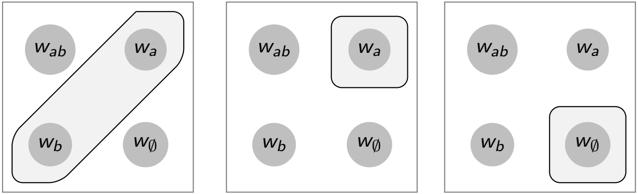
(a) Verifier (b) Zero-model (c) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\} \models (a \vee b)$, because we can find substates supporting each disjunct: $\{w_a\}$ itself, supporting a, and $\emptyset$, vacuously supporting b
- $\{w_a\}$ is then a **zero-model** for $(a \vee b)$, a model which verifies the formula by virtue of an empty witness
- Core effect of **neglect-zero enrichment**: disallow such zero-models
- Different implementations possible in BSML

Neglect-zero effects in BSML: core idea

- s supports a **neglect-zero enriched disjunction** iff s is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models$ enriched$(a \vee b)$ (b) $\not\models$ enriched$(a \vee b)$ (c) $\not\models$ enriched$(a \vee b)$

- A neglect-zero enriched disjunction requires both disjuncts to be live possibilities
- Aloni 2021 defined neglect-zero enrichment in terms of non-emptiness atom (NE) from team logic

Neglect-zero effects in BSML: implementation Aloni 21

- Non-emptiness atom (NE): NE requires the supporting state to be non-empty:

$$M, s \models \mathrm{NE} \quad \text{iff} \quad s \neq \emptyset$$

- Pragmatic enrichment function: Pragmatically enriched formula $[\alpha]^+$ comes with the requirement to satisfy NE distributed along each of its subformulas:

$$\begin{array}{rcl} [p]^+ & = & p \wedge \mathrm{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \mathrm{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \mathrm{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \mathrm{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \mathrm{NE} \end{array}$$

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to positive disjunctions:
  - $\mapsto$ we derive FC effects (for pragmatically enriched formulas);
  - $\mapsto$ pragmatic enrichment vacuous under single negation.

Neglect-zero effects in BSML: predictions Aloni 2021

After pragmatic enrichments

- We derive both wide and narrow scope FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (with restrictions)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before pragmatic enrichments

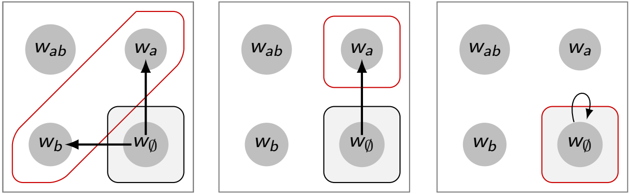
- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture infelicity of epistemic contradictions (Yalcin, 2007) by putting team-based constraints on accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

Neglect-zero effects in BSML: illustrations

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for all worlds in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:



(d) Verifier (e) Zero-model (f) Falsifier

Figure: Models for $\Diamond(a \vee b)$.

- **Negation** facts follow from adopted bilateralism (we validate De Morgan laws and $\neg\neg$-elimination):
  - Adding NE vacuous under single negation:

$$\neg(\alpha \wedge \mathrm{NE}) \equiv \neg\alpha \vee \neg\mathrm{NE} \equiv \neg\alpha \vee \bot \equiv \neg\alpha$$

  - Adding NE non-vacuous under double negation:

$$\neg\neg(\alpha \wedge \mathrm{NE}) \equiv \alpha \wedge \mathrm{NE}$$

Neglect-zero effects in BSML: predictions Aloni 2021

- Two more predictions:
  - **Modal disjunction**: $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (if R is state-based)
  - **Negative** FC: $[\neg\Box(\alpha \wedge \beta)]^+ \not\models \Diamond\neg\alpha \wedge \Diamond\neg\beta$

- In BSML logically equivalent sentences can have different neglect-zero effects, i.e., these effects are detachable:

$$\Diamond(\neg\alpha \vee \neg\beta) \quad \equiv \quad \neg\Box(\alpha \wedge \beta)$$

But:

$$[\Diamond(\neg\alpha \vee \neg\beta)]^+ \quad \models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$
$$[\neg\Box(\alpha \wedge \beta)]^+ \quad \not\models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$

Status of Modal disjunction and Negative FC debated in the literature

Modal disjunction and Negative FC

- **Experimental research**: modal disjunction and negative FC inferences exist but appear to be less available than positive FC:

(22) Negative FC (Marty et al., 2021)

a. It is not required that Mia buys both apples and bananas $\rightsquigarrow$ It is not required that Mia buys apples and that Mia buys bananas

b. $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$ $\qquad (\equiv \Diamond\neg\alpha \wedge \Diamond\neg\beta)$

(23) Modal disjunction (Tieu et al., 2019)

a. Angie bought the boat or the car $\rightsquigarrow$ Angie might have bought the boat and might have bought the car

b. $\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$

- BSML$^+$: BSML + global pragmatic enrichment

$$\alpha \models_{BSML^+} \beta \text{ iff } [\alpha]^+ \models_{BSML} [\beta]^+$$

- Mismatch between BSML$^+$ and experimental findings:

			BSML$^+$
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**strong**	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**weak**	-
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	**weak***	+

Comparison with two recent approaches

- Goldstein 2019: FC inferences derived by adding a homogeneity presupposition to the meaning of
  - possibility modal [alternative-based account, Gold19A]
  - disjunction [dynamic account, Gold19B]
- Bar-Lev & Fox 2020: FC inference derived by application of an exhaustivity operator (which includes alternatives on top of negating all the innocently excludable ones) [BLF20]

		BSML$^+$	Gold19A	Gold19B	BLF20
Positive FC	**strong**	+	+	+	+
Negative FC	**weak**	-	-	-	+
Modal Disjunction	**weak***	+	-	+	-

Table: Comparison BSML$^+$ and alternative approaches

- Gold19A seems the best option but needs to be supplemented with a theory deriving weak inferences;
- NEXT: neglect-zero can explain both weak and strong inference patterns but we need to
  1. Assume that neglect-zero effects can be conventionalised;
  2. Give a better formalisation of purely pragmatic (weak) neglect-zero effects.

Modelling neglect-zero effects: different implementations

- More ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function $[\ ]^+$ defined in terms of $\textsc{ne}$ $\mapsto$ BSML$^+$
  - Model-theoretically, by ruling out $\emptyset$ from the set of possible states $\mapsto$ BSML*
- Both implementations derive:
  - $\mapsto$ $\textsc{fc}$ effects (narrow and wide scope $\textsc{fc}$, the latter with restrictions);
  - $\mapsto$ cancellations of $\textsc{fc}$ effects under negation (dual prohibition).
- But empirical and conceptual differences:
  - Only BSML* predicts Negative $\textsc{fc}$: $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$
  - Only in BSML, where $[\ ]^+$ and $\emptyset$ are parts of the building blocks, locality and suspension of neglect-zero effects can be modeled
- Conjecture: neglect-zero can cause two kinds of effects:
  - (i) cancellable *global* non-detachable effects (modelled by BSML*);
  - (ii) more robust effects triggered by the conventional meaning of certain expressions (modelled by obligatory *local* applications of $[\ ]^+$).

Purely pragmatic neglect-zero effects: BSML*

- BSML*: like BSML, but $\emptyset$ is not among the possible states;
- **Fact:** Let α, β be classical positive formulas. Then

$$\alpha \models_{BSML^*} \beta \text{ iff } \alpha \models_{BSML^+} \beta$$

- But this does not hold in general. In BSML*, FC inferences generated also for negative conjunctions ($\Rightarrow$ Negative FC):

$$\begin{array}{lcl} \Diamond\neg(\alpha \wedge \beta) & \models_{BSML^*} & \Diamond\neg\alpha \wedge \Diamond\neg\beta \\ \neg\Box(\alpha \wedge \beta) & \models_{BSML^*} & \neg\Box\alpha \wedge \neg\Box\beta \end{array}$$

- Conjecture (i): BSML* characterises purely pragmatic neglect-zero effects: not the result of costly syntactic enrichments but deriving from the omission of the empty set (**low cost pragmatics**)

			BSML$^+$	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**s**	+	+
Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	**s**	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	+
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	**w***	+	+

Conventionalised neglect-zero enrichments

- Conjecture (ii): neglect-zero enrichments can be(come) part of the conventional meaning of certain expressions, e.g.
  - **universal quantifiers** leading to existential import presuppositions

    (24) Every square is black $\rightsquigarrow$ Some square is black

  - **epistemic indefinites** leading to obligatory ignorance effects

    (25) *Irgendjemand* hat angerufen $\rightsquigarrow$ Speaker doesn't know who

  - **modal verbs** leading to obligatory FC effects

    (26) You may eat the cake or the ice-cream $\rightsquigarrow$ You may eat the cake

- Working hypothesis these conventionalisations are not lexical stipulations but rather emerge from the urge to communicate in an effective but learnable way.
- Only our original BSML, with $\emptyset$ and $[\,]^+$ among its building blocks, can model such local conventionalisations of neglect-zero effects

Conventionalised neglect-zero enrichments in modals

- BSML$^\Diamond$: possibility and necessity modal verbs trigger $[\,]^+$-enrichment in their prejacents as part of their conventional meaning
- BSML$^\Diamond$ predicts a contrast between
  - positive FC (valid) vs
  - negative FC & modal disjunction (not valid)

  which gives in combination with BSML* a better match with experimental findings:

			BSML$^\Diamond$	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**s**	$+$	$+$
Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	**s**	$+$	$+$
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	$+$
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	**w***	-	$+$

BSML$^\Diamond$ $+$ BSML*: same predictions as Gold19A $+$ BSML*, but with neglect-zero as unique source for both semantic and pragmatic effects.

What about overt FC cancellations?

- Overt FC cancellation:

(27) You may either eat the cake or the ice-cream, I don't know which
$\not\rightsquigarrow$ You may eat the cake

- Prediction of BSML$^\diamond$: all cases of overt FC cancellations involve a wide scope configuration:
  1. Narrow scope FC: $\diamond[\alpha \vee \beta]^+ \models \diamond[\alpha]^+ \wedge \diamond[\beta]^+$
  2. Wide scope FC: $\diamond[\alpha]^+ \vee \diamond[\beta]^+ \not\models \diamond[\alpha]^+ \wedge \diamond[\beta]^+$
- Sluicing in (28) arguably triggers wide scope disjunctions (Fusco 2018, Pinton 2021):

(28) You may either eat the cake or the ice-cream, I don't know which (you may eat). [wide, –fc]

- Wide scope configuration also required for (29) (Kaufmann 2016):

(29) You may either eat the cake or the ice-cream, it depends on what John has taken. [wide, –fc]

- Wide scope FC captured as weak/global neglect-zero effect:
  - BSML*: $\diamond\alpha \vee \diamond\beta \models_{BSML^*} \diamond\alpha \wedge \diamond\beta$ [if R is indisputable]

Global suspension of neglect-zero effects: BSML$^{\emptyset}$

- Despite their cognitive cost, zero-models are not always neglected.
- In logico-mathematical reasonings, neglect-zero effects are globally suspended:

(30) A. Therefore, A or B.

(31) If A then B. Therefore, if not B then not A.

- Global suspension modeled by BSML$^{\emptyset}$ = BSML*+ $\emptyset$ (BSML$\setminus$NE)

		BSML$^{\emptyset}$	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	-	+
Addition	$\alpha \models \alpha \vee \beta$	+	-
Contraposition	$\alpha \models \beta \;\Rightarrow\; \neg\beta \models \neg\alpha$	+	-

- In BSML$^{\emptyset}$ (= classical logic), $\emptyset$ plays an essential role:

  The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (Alfred North Whitehead, quoted by Nieder 2016)

- Final conjecture: at least in part, divergence between human and logico-mathematical reasoning might be due to a neglect-zero tendency.

The resulting picture

- A pluralism of systems which can be used to model interpretation strategies & reasoning styles people may adopt in different circumstances:
  1. $\text{BSML}^{\emptyset}$: modelling logical-mathematical reasoning where neglect-zero effects are obviated;
  2. BSML^{*}: modelling global purely pragmatic (non-detachable) neglect-zero effects;
  3. $\text{BSML}^{\Diamond}$: modelling local conventionalisations of neglect-zero effects;
  4. ...
- Experimentally testable predictions arising from these conjectures

			$\text{BSML}^{\emptyset}$	$\text{BSML}^{\Diamond}$	BSML^{*}
NS FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	s	-	+	+
Dual prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	s	+	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	w	-	-	+
Modal disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	w*	-	-	+
WS FC	$\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	?	-	-	+

Table: Comparison $\text{BSML}^{\emptyset}$, $\text{BSML}^{\Diamond}$ and BSML^{*}.

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but, when applied to FC, empirically inadequate
- My proposal: FC as neglect-zero effect in BSML
  - Aloni 2021: Classical logic (NE-free fragment) + neglect-zero (NE) $\Rightarrow$ FC and related inferences
  - Different implementations of (suspension of) neglect-zero effects:
    - BSML* vs BSML$^\Diamond$ vs BSML$^\emptyset$
- Related (future) research:
  - Logic: proof theory (Anttila (MoL 2021), Yang, MA); bimodal perspective (Baltag, van Benthem, Bezhanishvili, MA); QBSML (MA & van Ormondt);
  - Language: FC cancellations (Pinton (MoL 2021), Hui (MoL 2021)); modified numerals (MA & van Ormondt); indefinites (MA & Degano); acquisition (children's conjunctive strengthening of disjunction); experiments.

Appendix

Bilateral State-Based Modal Logic (BSML)

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$

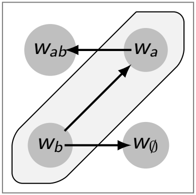
where $p \in A$.

Models and States

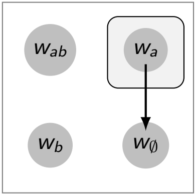
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model [$s \neq \emptyset$ in BSML*]

Examples

for $A = \{a, b\}$



(a) $\not\models a$; $\models \Diamond a$



(b) $\models a$; $\not\models \Diamond a$

Semantic clauses

$[M = \langle W, R, V \rangle;\ s, t, t' \subseteq W]$

$$
\begin{array}{rcl}
M, s \models p & \text{iff} & \forall w \in s : V(w,p) = 1 \\
M, s \dashv p & \text{iff} & \forall w \in s : V(w,p) = 0 \\[6pt]
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\[6pt]
M, s \models \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\[6pt]
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\[6pt]
M, s \models \Diamond\phi & \text{iff} & \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \forall w \in s : M, R[w] \dashv \phi \\[6pt]
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R[w] = \{v \in W \mid wRv\}$

Box

- $\Box\phi := \neg\Diamond\neg\phi$

$$\begin{array}{lll} M, s \models \Box\phi & \text{iff} & \text{for all } w \in s : M, R[w] \models \phi \\ M, s \dashv \Box\phi & \text{iff} & \text{for all } w \in s : \text{there is a } t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \dashv \phi \end{array}$$

where $R[w] = \{v \in W \mid wRv\}$

Logical consequence

- $\phi \models \psi$ iff for all M, s : $M, s \models \phi \ \Rightarrow \ M, s \models \psi$

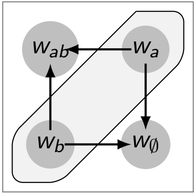
Pragmatic enrichment

For NE-free α, $[\alpha]^+$ defined as follows:

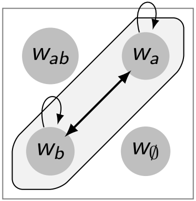
$$\begin{array}{rcl} [p]^+ & = & p \wedge \text{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \text{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \text{NE} \end{array}$$

Team-sensitive constraints on accessibility relation

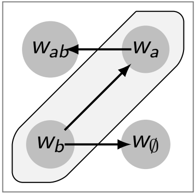
- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
  $\mapsto$ all worlds in s access exactly the same set of worlds
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$
  $\mapsto$ all and only worlds in s are accessible within s



(c) indisputable



(d) state-base (and so also indisputable)



(e) neither

Deontic vs epistemic modal

- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Applications: epistemic free choice

Narrow scope and wide scope FC

1. $[\Diamond(a \vee b)]^+ \models \Diamond a \wedge \Diamond b$
2. $[\Diamond a \vee \Diamond b]^+ \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Epistemic modals

- R is state-based, therefore always indisputable:

  (32) He might either be in London or in Paris. [+fc, narrow]

  (33) He might be in London or he might be in Paris. [+fc, wide]

- $\Rightarrow$ narrow and wide scope FC always predicted for pragmatically enriched epistemics

Applications: deontic free choice

Narrow scope and wide scope FC

1. $[\Diamond(a \vee b)]^+ \models \Diamond a \wedge \Diamond b$
2. $[\Diamond a \vee \Diamond b]^+ \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Deontic modals

- R may be indisputable if speaker is knowledgable (e.g. in performative uses)
- Predictions:
  - $\Rightarrow$ narrow scope FC always predicted for enriched deontics
  - $\Rightarrow$ wide scope FC only if speaker knows what is permitted/obligatory

Deontic FC: comparison with localist view

- Our proposal vs Fox (2007)

	NS+K	NS¬K	WS+K	WS¬K
MA	yes	yes	yes	no
Fox (2007)	yes	no	no	no

K $\mapsto$ speaker knows what is permitted/obligatory;
NS $\mapsto$ narrow scope FC; WS $\mapsto$ wide scope FC.

- Our predictions confirmed by pilot experiment (Cremers et al. 2017)
- Speaker knowledge has effect on FC inference only in wide scope configurations:

(34) We may either eat the cake or the ice-cream. [narrow, +fc]

(35) Either we may eat the cake or the ice-cream. [wide, +/–fc]

Position of *either* favors a narrow scope interpretation in (34), while it forces a wide scope interpretation in (35) (Larson 1985)