Neglect-zero and no-split: cognitive biases at the semantic-pragmatic interface

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Free Choice Inferences: Theoretical and experimental approaches
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Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic

Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments

Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic/cognitive factors and the additional inferences which arise from their interaction

Novel hypothesis: *neglect-zero* tendency (and *no-split*) as crucial pragmatic/cognitive factors

Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Nihil team
MA, Anttila, Knudstorp, Degano, **Klochowicz**, Ramotowska, **Sbardolini**
Non-classical inferences

Free choice (FC)

(1) \( \Diamond (\alpha \lor \beta) \leadsto \Diamond \alpha \land \Diamond \beta \)

(2) Deontic FC inference [Kamp 1973]
   a. You may go to the beach or to the cinema.
   b. \( \leadsto \) You may go to the beach and you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]
   a. Mr. X might be in Victoria or in Brixton.
   b. \( \leadsto \) Mr. X might be in Victoria and he might be in Brixton.

Ignorance

(4) The prize is in the attic or in the garden \( \leadsto \) speaker doesn’t know where

(5) ? I have two or three children. [Grice 1989]

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC inferences analysed as conversational implicatures; grammatical implicatures; semantic entailments; . . .
Novel hypothesis: neglect-zero

- FC and ignorance inferences are \[ \neq \text{semantic entailments} \]
  - Not the result of Gricean reasoning \[ \neq \text{conversational implicatures} \]
  - Not the effect of applications of covert grammatical operators \[ \neq \text{scalar implicatures} \]

- But rather a consequence of something else speakers do in conversation, namely,

**Neglect-Zero**

when interpreting a sentence speakers create mental structures representing reality\(^1\) and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (zero-models)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets \[ \text{[Nieder 2016, Bott et al, 2019]} \]

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Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.
   a. Verifier: [◼,◼,◼]
   b. Falsifier: [◼,□,◼]
   c. Zero-models: [ ]; [△,△,△]; [◇,▲,◇]; [▲,▲,▲]

(7) Less than three squares are black.
   a. Verifier: [◼,□,◼]
   b. Falsifier: [◼,◼,◼]
   c. Zero-models: [ ]; [△,△,△]; [◇,▲,◇]; [▲,▲,▲]; [□,□,□]

- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (less vs more) [Bott et al., 2019]
  - existential import & connexive principles operative in Aristotelian logic (every A is B ⇒ some A is B; not (if not A then A)) [MA, 2024]

- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences
Novel hypothesis: neglect-zero

Illustrations

(8) It is raining.
   a. Verifier: \[
   \]
   b. Falsifier: \[
   \]
   c. Zero-models: none

(9) It is snowing.
   a. Verifier: \[
   \]
   b. Falsifier: \[
   \]; \[
   \]; \ldots
   c. Zero-models: none

(10) It is raining or snowing.
   a. Verifier: \[
   \]
   b. Falsifier: \[
   \]
   c. Zero-models: \[
   \]; \[
   \]; \[
   \]

▶ Two models in (10-c) are zero-models because they verify the sentence by virtue of an empty witness for one of the disjuncts
▶ Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded
Novel hypothesis: no-split

A closer look at the disjunctive case

(11) It is raining or snowing.
    a. Verifier: $[\text{rain} \mid \text{snow}]$ [⇐ “split” state]
    b. Falsifier: $[\text{sun}]$
    c. Zero-models: $[\text{rain}]; [\text{snow}]$

- The “split” verifier in (11-a) involves the entertainment of two alternatives, also arguably a cognitively difficult operation

- We conjecture that the ability to split states is acquired late
  $\Rightarrow$ NO-SPLIT HYPOTHESIS

- The combination of neglect-zero and no-split can explain non-classical inferences observed in pre-school children
No-split and the acquisition of ‘or’

▶ **Basic data:** some pre-school children interpret *or* as *and* [e.g., Singh *et al* 2016, Cochard 2023, Bleotu *et al* 2024]:

(12) The boy is holding an apple or a banana = The boy is holding an apple and a banana

\[(\alpha \lor \beta) = (\alpha \land \beta)\]

(13) Every boy is holding an apple or a banana = Every boy is holding an apple and a banana

\[\forall x(\alpha \lor \beta) = \forall x(\alpha \land \beta)\]

(14) Liz can buy a croissant or a donut = Liz can buy a croissant and a donut

\[\Diamond(\alpha \lor \beta) = \Diamond(\alpha \land \beta)\]

▶ **Two different explanations:**

▶ **Singh *et al*:** derive \(\alpha \land \beta\) from \(\alpha \lor \beta\) as a scalar implicature using

\[\text{exh-ALT} = \{\alpha \land \neg \beta, \beta \land \neg \alpha\}\]

[or, alternatively, by innocent inclusion]

\[\Leftrightarrow\] children can compute scalar implicatures and can exhaustify alternatives, but don’t have access to lexical alternatives

▶ **Nihil:** beside neglecting zero-models, children further lack the ability to split states, i.e. have difficulties in engaging with alternative epistemic possibilities, in picturing different ways the world might be.
**BSML: teams and bilateralism**

- **Team semantics:** formulas interpreted wrt a set of points of evaluation (a team) rather than single ones  
  
  [Hodges 1997; Väänänen 2007]

**Classical vs team-based modal logic**

- Classical modal logic:  
  
  \[ M = \langle W, R, V \rangle \]  
  
  (truth in worlds)

- Team-based modal logic:  
  
  \[ M, t \models \phi, \text{ where } t \subseteq W \]

**Bilateral state-based modal logic (BSML)**

- Teams \( \leftrightarrow \) information states  
  
  [Dekker93; Groenendijk\(^+\)96; Ciardelli\(^+\)19]

- Assertion & rejection conditions modelled rather than truth  
  
  \[ M, s \models \phi, \text{ “} \phi \text{ is assertable in } s \text{”, with } s \subseteq W \]

  \[ M, s \models \phi, \text{ “} \phi \text{ is rejectable in } s \text{”, with } s \subseteq W \]

- Neglect-zero tendency modelled by \( \text{NE} \)  
  
  [Yang & Väänänen 2017]

- BSML\(^F\): No-split modelled via a flattening operator \( F \)  
  
  [Punčochář 2024]
BSML$^F$: Classical Modal Logic + NE + F

Language

$$\phi ::= p \mid \neg \phi \mid \phi \lor \phi \mid \phi \land \phi \mid \Box \phi \mid \text{NE} \mid \text{F}$$

Bilateral team semantics

$$[M = \langle W, R, V \rangle \& s, t, t' \subseteq W]$$

$$M, s \models p$$ iff for all $w \in s : V(w, p) = 1$$

$$M, s \models \neg \phi$$ iff $M, s \models \phi$$

$$M, s \models \phi \lor \psi$$ iff there are $t, t' : t \cup t' = s \& M, t \models \phi \& M, t' \models \psi$$

$$M, s \models \phi \land \psi$$ iff $M, s \models \phi \& M, s \models \psi$$

$$M, s \models \Box \phi$$ iff for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset \& M, t \models \phi$$

$$M, s \models \text{NE}$$ iff $s \neq \emptyset$ [where $R[w] = \{ v \in W \mid wRv \}$]

$$M, s \models \text{F} \phi$$ iff for all $w \in s : M, \{ w \} \models \phi$$
Neglect-zero effects in BSML: split disjunction

- A state $s$ supports a disjunction $\phi \lor \psi$ iff $s$ is the union of two substates, each supporting one of the disjuncts.

![Diagram showing verifiers and falsifiers for disjunctions](image)

Figure: Models for $(a \lor b)$.

- $\{w_a\}$ verifies $(a \lor b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset \& M, \emptyset \models b \quad \rightarrow \text{zero-model}$

- Main idea: define neglect-zero enrichments, $[\ ]^+$, whose core effect is to rule out such zero-models.

- Implementation: $[\ ]^+$ defined using $\text{NE} (s \models \text{NE} \iff s \neq \emptyset)$, which models neglect-zero in the logic.
Neglect-zero effects in BSML: enriched disjunction

- $s$ supports an enriched disjunction $[\phi \lor \psi]^+$ iff $s$ is the union of two non-empty substates, each supporting one of the disjuncts

\[ [a \lor b]^+ \]

\[ [a \lor b]^+ \]

\[ [a \lor b]^+ \]

- An enriched disjunction requires both disjuncts to be live possibilities

(15) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to positive disjunctions $[\ ]^+$-enrichment vacuous under single negation.
More no-zero verifiers for enriched disjunction

\[ \vdash \neg (a \land b) \]

\[ \not \vdash \neg (a \land b) \]

\[ \not \vdash \neg \Box a \]

\[ \vdash \neg \Box a \]

Figure: Models for enriched \([a \lor b]^+\).
Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \lor b$:

(a) scalar

(b) no-uncertain

(c) no-split

- Two components of full ignorance (‘speaker doesn’t know which’):\(^2\)

(16) **It is raining or it is snowing** ($\alpha \lor \beta$) \(\sim\)

a. **Uncertainty**: $\neg \Box_e \alpha \land \neg \Box_e \beta$

b. **Possibility**: $\Diamond_e \alpha \land \Diamond_e \beta$ \quad (equiv $\neg \Box_e \neg \alpha \land \neg \Box_e \neg \beta$)

- **Fact**: Only possibility derived as neglect-zero effect:

- $\{w_{ab}, w_a\} \models \Diamond_e a \land \Diamond_e b$, but $\not\models \neg \Box_e a$

- $\{w_{ab}, w_a\}$: a no-zero model supporting possibility but neither uncertainty nor scalar implicature ($\not\models \neg (a \land b)$)

Two derivations of full ignorance

1. Neo-Gricean derivation [Sauerland 2004]
   (i) Uncertainty derived through quantity reasoning

   (17) \( \alpha \lor \beta \)  
   Assertion

   (18) \( \neg \Box_e \alpha \land \neg \Box_e \beta \)  
   Uncertainty (from quantity)

   (ii) Possibility derived from uncertainty and quality about assertion

   (19) \( \Box_e (\alpha \lor \beta) \)  
   Quality about assertion

   (20) \( \Rightarrow \Diamond_e \alpha \land \Diamond_e \beta \)  
   Possibility

2. Nihil derivation

   (i) Possibility derived as neglect-zero effect

   (21) \( \alpha \lor \beta \)  
   Assertion

   (22) \( \Diamond_e \alpha \land \Diamond_e \beta \)  
   Possibility (from neglect-zero)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (23) \( \neg (\alpha \land \beta) \)  
   Scalar implicature

   (24) \( \Rightarrow \neg \Box_e \alpha \land \neg \Box_e \beta \)  
   Uncertainty
Neglect-zero effects in BSML: possibility vs uncertainty

Comparison with competing accounts

- Neo-Gricean vs Nihil predictions
  - **Neo-Gricean**: No possibility without uncertainty
  - **Nihil**: Possibility derived independently from uncertainty

Experimental study

- Experimental findings in agreement with Nihil predictions [Degano et al 2023]
  - Using adapted mystery box paradigm, compared conditions in which
    - both uncertainty and possibility are false [zero-model]
    - uncertainty false but possibility true [no-zero, no-uncertain model]
  - Less acceptance when possibility is false (95% vs 44%)
- Evidence that possibility can arise without uncertainty
Neglect-zero and no-split

- More no-zero verifiers for \( a \lor b \):
  - (d) scalar
  - (e) no-uncertain
  - (f) no-split

\[ \{w_{ab}\} \text{ is a no-split verifier for the disjunction: no alternatives entertained;} \]

- Conjecture: only no-split verifiers accessible to ‘conjunctive’ pre-school children [Klochowicz, Sbardolini, MA, 2024]
- Combination of no-split and no-zero gives us conjunctive or
- Implementation: uses flattening operator \( F \)
  \[ M, s \models F\phi \text{ iff for all } w \in s : M, \{w\} \models \phi \]

Flattening \( \mapsto \) formulas always interpreted wrt to singleton states
Figure: Combination of no-split and no-zero gives us conjunctive or

(25) It is raining or snowing.

a. No-zero & split: $[\text{weather} | \text{weather} \text{ weather}]$ [adult-like]
b. Zero and no-split: $[\text{weather} \text{ weather}]$ [logician]
c. No-zero & no-split: $[\text{weather} + \text{weather}]$ ['conjunctive’ children]
No-split: some predictions

(26) a. \([F(\alpha \lor \beta)]^{+/*} \equiv \alpha \land \beta\]

b. \([\forall x F(\alpha \lor \beta)]^{+/*} \equiv \forall x (\alpha \land \beta)\]

c. \(\diamond F(\alpha \lor \beta)]^{+/*} \equiv \diamond (\alpha \land \beta)\]

d. \([-F(\alpha \lor \beta)]^{+/*} \equiv \neg \alpha \land \neg \beta\]

e. \([-F(\alpha \land \beta)]^* \equiv \neg \alpha \land \neg \beta, \text{ but } [-F(\alpha \land \beta)]^+ \not\equiv \neg \alpha \land \neg \beta\]

▶ Two ways to model neglect-zero effects:
  ▶ Syntactically, via pragmatic enrichment function \([ \ ]^+\) defined in terms of \(\text{NE} \mapsto \text{BSML}^+\)
  ▶ Model-theoretically, by ruling out \(\emptyset\) from the set of possible states \(\mapsto \text{BSML}^*\)

▶ Both implementations derive:
  \(\mapsto \text{FC effects (narrow and wide scope FC, dual prohibition, etc);}\)
  \(\mapsto \text{conjunctive or in combination with flattening (26-a-d).}\)

▶ But only BSML* predicts
  ▶ \textbf{Negative FC:} \(\neg \Box (\alpha \land \beta) \sim \neg \Box \alpha \land \neg \Box \beta\) \[\text{Marty et al}\]
  ▶ \textbf{Homogeneity effects} in combination with \(F\) (26-e) \[\text{Sbardolini23}\]

▶ Only in BSML\(^+\), \(\emptyset\) is part of the building blocks (natural to assume BSML* for “conjunctive” children who plausibly do not access \(\emptyset\))
Two views

- Two explanations of conjunctive ‘or’ in pre-school children:
  - **Grammatical view**: conjunctive children can compute implicatures but do not have access to scalar alternatives (or < and);
  - **Nihil**: conjunctive behaviour derives from the combination of two cognitive bias: no-zero and no-split.

<table>
<thead>
<tr>
<th></th>
<th>conjunctive or</th>
<th>inclusive or</th>
<th>exclusive or</th>
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<tr>
<td>Grammatical</td>
<td>exh-alt [✓]</td>
<td>exh-alt [no]</td>
<td>scalar-alt [✓]</td>
</tr>
<tr>
<td>Nihil</td>
<td>zero [no] &amp; split [no]</td>
<td>split [✓]</td>
<td>split [✓] &amp; scalar reasoning [✓]</td>
</tr>
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- Two different acquisition patterns:
  - **Grammatical view**: inclusive or < conjunctive or < exclusive or
  - **Nihil**: conjunctive or < inclusive or < exclusive or
Conclusions

▶ **FC and related inferences**: a mismatch between logic and language
▶ **Grice’s insight**:
  ▶ stronger meanings can be derived paying more “attention to the nature and importance to the conditions governing conversation”
▶ **Nihil proposal**: stronger meanings consequences of cognitive biases
  ▶ FC and ignorance as neglect-zero effects
    
    Literal meanings (NE-free fragment) + cognitive factors (NE) 
    \[ \Rightarrow \text{FC & possibility inferences} \]
  
▶ **Conjunctive or** as no-zero + no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) \[ \Rightarrow \text{conjunctive or} \]

▶ **Implementation in BSML^F** (a team-based modal logic)
Collaborators & related (future) research

Logic
Proof theory (Anttila, Yang); expressive completeness (Anttila, Yang, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); Aristotelian logic in qBSML→ (MA);...

Language
FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Ramotowska, Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); acquisition (Sbardolini, Klochowicz);...

Thank You!³

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