A Semantics for Imperatives

Maria Aloni and Ivano Ciardelli

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Abstract

Imperatives like 'Post this letter or burn it!' or 'Take any card!' are most naturally interpreted as presenting a choice between different alternatives. The article proposes an account of choice-offering imperatives and of their non-standard logical properties based on the framework of inquisitive semantics.

1 Introduction

The law of propositional logic that states the deducibility of $\varphi \vee \psi$ from φ does not seem to be valid for imperatives (Ross' paradox, cf. Ross [1941]). The command (or request, advice, etc.) in (1-a) does not seem to imply (1-b), otherwise when told the former, I would be justified in burning the letter rather then posting it.

(1) a. Post this letter! $\not\Rightarrow$
b. Post this letter or burn it!

Intuitively the most natural interpretation of the second imperative is as one presenting a choice between two actions.

(2) a. Post this letter or burn it! $\Rightarrow$
b. You may post the letter and you may burn it.

Following Åquist [1965] (and Hamblin [1987]) we call these *choice-offering* imperatives. Another example of a choice-offering imperative is (3-a) with an occurence of Free Choice *any* which, somehow surprisingly, is licensed in this context:

(3) a. Take any card!
b. #John took/must take any card.

Like (1-b), this imperative is naturally interpreted as presenting a choice between different alternatives.

(4) a. Take any card! $\Rightarrow$
b. You may take card a, you may take card b, ...

Contrast (3-a) with “Take some card!”, which lacks this choice-offering interpretation.

Different views on the status of the free choice inferences in (2) and (4) can be defended: they might be taken to be purely pragmatic implicatures or to have the status of semantic entailments. The fact that free choice inferences are not embeddable under negation seems to argue in favor of the former view. Imperative (5-a) cannot mean ‘Post this letter or burn it, but you may not do both’, and (5-b) cannot mean ‘Take a card but you may not choose which’.

(5) a. Don’t post this letter or burn it!
 b. Don’t take any card!

On the other hand, in positive contexts, these free choice inferences do not seem to be cancelable as illustrated by the following two examples from Aloni [2007] (which are modified versions of examples from Hamblin [1987] and Mastop [2005] respectively). These examples seem to provide evidence in favor of a semantic account.

(6) GRANDMA: Take any card!
 KID GETS UP TO PICK A CARD.
 GRANDMA: ??? Don’t you dare take the ace!

(7) MOTHER: Do your homework or help your father in the kitchen!
 SON GOES TO THE KITCHEN.
 FATHER: Do your homework!
 SON: But, mom told me I could also help you in the kitchen!

In this article we will assume that free choice inferences in choice-offering imperatives are matters of entailment. This will yield a ready account of examples (6) and (7), but also of Ross’ paradox in (1), and of the contrast in grammaticality illustrated in (3). Furthermore, as it will be clear later on, we will derive the right interpretation for the negative imperatives in (5).

Recently a number of dynamic logics for imperatives have been proposed to account for (1) and (7) [e.g. Mastop, 2005, Veltman, 2009]. On these systems an imperative sentence performatively changes the to-do-list of some agent. These approaches are promising in that they derive (1) and (7) from the performative nature of imperatives. They only discuss the propositional case though, and it is hard, if not impossible, to extend them to the first-order case to account for (3) and (6).

As far as we know, the only attempt to capture both (1) and (3) is Aloni [2007]. On that account, *any* and *or* are treated as operators which introduce sets of propositional alternatives. The imperative operator is then analyzed as a quantifier over these sets of alternatives. Choice-offering imperatives are distinguished from basic imperatives in that they involve genuine sets of propositional alternatives. The logic we will present in this article share these characteristics with Aloni [2007], but, as we will argue, it improves on it both empirically and conceptually.

First of all, Aloni [2007] generates alternative propositions via a dynamic semantics [e.g. Dekker, 2002] supplemented with a mechanism of propositional quantification [Fine, 1970]. In the present article, instead, we use the framework of first-order inquisitive semantics[1] developed in Ciardelli [2009]. The latter formalism has many advantages from a logical point of view as it will become clear later on, but it has also an important empirical motivation. In Aloni, alternative propositions are generated by dynamically active existential propositional quantifiers in interaction with disjunction and existential individual quantification. One of the predictions of that system is that within a *static* operator, i.e. an operator that blocks anaphoric links between a term that occurs in its scope and a pronoun outside of it [cf. Groenendijk and Stokhof, 1991], no alternatives are generated, and, therefore, no choice-offering readings can arise. Negation and the universal quantifiers are both examples of static operators. Aloni's prediction, however, is only borne out in the case of negation. Negative imperative are never choice-offering (cf. example (5)), but universal one might be. For example, (8) grants for each letter the permission to post it or burn it.

(8) For every letter, post it or burn it!

In inquisitive semantics, where the potential to generate alternatives is not related to the dynamic nature of the scoping operator, this problem does not arise.

Another important difference between the present account and Aloni's system concerns the nature of the predicted free choice inference. In Aloni [2007] choice-offering disjunctive and existential imperatives grant the permission to freely choose one of the relevant possibility and execute it:

(9) a. A or $B! \models \Diamond A \wedge \Diamond B$
b. Any $A! \models \forall x \Diamond A(x)$

On the present account, instead, choice offering imperatives will have the stronger entailment that each possibility may be executed *in isolation*:

(10) a. A or $B! \models \Diamond(A \wedge \neg B) \wedge \Diamond(B \wedge \neg A)$
b. Any $A! \models \forall x \Diamond \forall y (A(y) \leftrightarrow x = y)$

As arguments in favor of (10) consider the following two examples.

Seminar Consider the following disjunctive imperative (attributed to an anonymous reviewer in Aloni [2007]):

(11) To pass the seminar, write a paper, give a presentation, or take an oral exam.

[1] Inquisitive semantics is a young but very active area of research: see among others [Mascarenhas, 2009, Groenendijk, 2009, Ciardelli and Roelofsen, 2011, Groenendijk and Roelofsen, 2010], whose focus is, however, restricted to a propositional language.

Assume one gets credit for an oral exam (obligatory) combined with either giving a presentation or writing a paper. Imperative (11) would be misleading in the given scenario. However, Aloni [2007] predicts the sentence to be acceptable. On our account, where the stronger (10-a) is valid, the deviance of (11) will be accounted for.

The next scenario from Menéndez-Benito [2005] illustrates the same point for *any*-imperatives.

Canasta One of the rules of the card game Canasta is: when a player has two cards that match the top card of the discard pile, she has two options: (i) she can take all the cards in the discard pile or (ii) she can take no card from the discard pile (but take the top card of the regular pile instead). Consider now the following imperative:

(12) Take any card from the discard pile!

Intuitively, (12) would not count as command (or request, advice, etc.) to choose option (i), contrary to what Aloni predicts. Again, by validating the stronger (10-b), the present system will avoid this problem.

The article is structured as follows. The next section presents a semantics for imperatives. Section 3 extends the language with modal operators and relates imperatives to deontic $\Box$ and $\Diamond$. Section 4 draws some conclusions and section 5 indicates further lines of research.

2 Imperatives

2.1 A language for imperatives

First of all, in order to set up our semantics we need to specify a formal language for imperatives. Of course, many choices are possible: we shall use a simple first-order language $\mathcal{L}$. The idea is that atomic formulas name basic imperatives, and that complex imperatives may be obtained by means of the Boolean connectives $\neg, \wedge, \vee$ and the standard quantifiers $\exists, \forall$. Thus, a nullary predicate symbol p will stand for basic imperatives such as 'run' or 'sleep', a unary symbol $P(x)$ for imperatives with a complement, such as 'throw x' or 'kiss x', etcetera.

Clearly, imperatives cannot be said to be true or false in a certain state of affairs: all we can say is whether or not an agent has complied with an imperative over a certain lapse of time. Thus, the usual first-order models for the language $\mathcal{L}$ will be called, and conceived of as *conducts*: the idea is that a model specifies which actions are executed (by a given agent over a certain lapse of time) and which ones are not, rather than which things do and do not hold in a certain state of affairs. For instance, the interpretation of a unary predicate 'kiss x' in a model will represent the set of individuals kissed by the agent in that conduct. For nullary predicate symbols, the model simply specifies whether or not they are executed in that conduct.

Thus, we read the classical satisfaction relation $M, g \models \varphi$ between a model a formula (relative to an assignment g) as "the conduct M complies with the imperative φ" or "φ is executed in the conduct M".

To make this precise, fix a domain D and an interpretation f^D of each function symbol in $\mathcal{L}$ (including constants), and call $\mathbb{D}$ the resulting structure. A $\mathbb{D}$-model is a first-order model for the language $\mathcal{L}$ based on $\mathbb{D}$. We denote by ω the set of all $\mathbb{D}$-models.

Definition 2.1 (Compliance set)**.** Relative to an assignment g into D, the *compliance set* $|\varphi|_g$ of an imperative φ is simply the set of $\mathbb{D}$-models which comply with the imperative:

$$|\varphi|_g = \{M \in \omega \mid M, g \models \varphi\}$$

2.2 Basic and choice imperatives

The most striking and common effect of an imperative is to specify an obligation, as in (13).

(13) Call Mark.

Additionally, as we saw in the introduction, imperatives may offer a choice, as in (14) and in (15). In this case, we speak of *choice-offering* or *choice* imperatives.

(14) Call Andrew or Mark.

(15) Call any of your friends.

For instance, (15) requires the hearer to call at least one friend and it grants the permission to choose a specific friend and call that friend and only that one. Similarly for (14), where the choice is between Mark and Andrew.

In general, we shall analyze choice imperatives as conveying the obligation to execute the imperative together with the permission to execute the imperative *in any possible way*. But what exactly are the *ways* in which an imperative may be executed? The following definition takes care of answering this question.

Definition 2.2 (Possibilities for an imperative)**.** The set $[\![\varphi]\!]_g$ of possibilities for an imperative relative to an assignment g is defined recursively as follows:

1. $[\![\varphi]\!]_g = \{|\varphi|_g\}$ if φ is atomic;
2. $[\![\neg\varphi]\!]_g = \omega - \bigcup[\![\varphi]\!]_g$;
3. $[\![\varphi \vee \psi]\!]_g = [\![\varphi]\!]_g \cup [\![\psi]\!]_g$;
4. $[\![\varphi \wedge \psi]\!]_g = \{s \cap t \mid s \in [\![\varphi]\!]_g \text{ and } t \in [\![\psi]\!]_g\}$;
5. $[\![\exists x\varphi]\!]_g = \bigcup_{d \in D}[\![\varphi]\!]_{g[x \mapsto d]}$;
6. $[\![\forall x\varphi]\!]_g = \{\bigcap_{d \in D} s_d \mid s_d \in [\![\varphi]\!]_{g[x \mapsto d]} \text{ for all } d \in D\}$.

The clauses can be read as follows: atoms denote basic imperatives; to execute $\neg\varphi$, one must not execute φ in any way; to execute $\varphi \vee \psi$, one has to execute either φ or ψ; to execute $\varphi \wedge \psi$, one has to execute both φ and ψ; to execute $\exists x\varphi$, one has to execute $\varphi[^d/_x]$ for some individual d; finally, to execute $\forall x\varphi$, one has to execute $\varphi[^d/_x]$ for all individuals d.

Notice that any imperative φ has at least one (possibly empty) possibility. Moreover, the following remark shows that the possibilities for an imperative exhaust its realization set: in other words, an imperative is executed iff it is executed in some of the ways we identified.

Remark 2.3. For any φ and g, $\bigcup[\![\varphi]\!]_g = |\varphi|_g$.

It follows from this observation that $[\![\neg\varphi]\!]_g = \omega - \{|\varphi|_g\} = \{|\neg\varphi|_g\}$. If an imperative has only one possibility we say that it is *basic*, otherwise we say that it is a *choice* imperative.

The following remark gives sufficient syntactic conditions on a formula to denote a basic imperative.

Remark 2.4. For any assignment g,

1. atomic imperatives and negations are basic relative to g;
2. if both φ and ψ are basic relative to g, so is $\varphi \wedge \psi$;
3. if φ is basic relative to $g[x \mapsto d]$ for all $d \in D$, then $\forall x\varphi$ is basic relative to g.

In particular, any imperative built up from atoms and negations by means of conjunction and the universal quantifier alone is basic. This means that the only sources of choice in the language are disjunction and the existential quantifier. As we shall see, these logical constants mirror the natural language free-choice items *or* and *any*.

Also, observe that double negation has the effect of collapsing all possibilities for an imperative into a single one, corresponding to the truth-set of the formula: $[\![\neg\neg\varphi]\!]_g = \{|\varphi|_g\}$. Thus, the effect of double negation is to erase the choice component specified by the imperative. We will thus call the formula $\neg\neg\varphi$ the *flattening* of φ, and denote if by $F\varphi$.

Realizations As soon as the domain D is finite and each element d has a name $\overline{d}$, possibilities admit a syntactic counterpart which we call *realizations*.

Definition 2.5 (Realizations of an imperative)**.** The set $\mathcal{R}(\varphi)$ of realizations of an imperative φ is defined recursively as follows:

1. $\mathcal{R}(\varphi) = \{\varphi\}$ if φ is atomic;
2. $\mathcal{R}(\neg\varphi) = \{\neg\varphi\}$;
3. $\mathcal{R}(\varphi \vee \psi) = \mathcal{R}(\varphi) \cup \mathcal{R}(\psi)$;

4. $\mathcal{R}(\varphi \wedge \psi) = \{\rho \wedge \sigma \mid \rho \in \mathcal{R}(\varphi) \wedge \sigma \in \mathcal{R}(\psi)\}$;

5. $\mathcal{R}(\exists x\varphi) = \bigcup\{\mathcal{R}(\varphi[^{\overline{d}}/_{x}]) \mid d \in D\}$;

6. $\mathcal{R}(\forall x\varphi) = \{\bigwedge_{d\in D} \rho_d \mid \rho_d \in \mathcal{R}(\varphi[^{\overline{d}}/_{x}])$ for all $d \in D\}$.

Note that realizations do not contain disjunctions or existential quantifiers, and thus they denote basic imperatives. Observe that even if the domain D is *not* finite, realizations are equally well-defined for all formulas which do not contain universal quantifiers.

The next lemma clarifies in what sense realizations are the syntactic counterpart of possibilities.

Proposition 2.6. If realizations are defined for φ, then for any g,

$$[\![\varphi]\!]_g = \{|\rho|_g \mid \rho \in \mathcal{R}(\varphi)\}$$

As a consequence, any imperative may be written as the disjunction of its realizations: if realizations are defined for φ, then $[\![\varphi]\!]_g = [\![\bigvee \mathcal{R}(\varphi)]\!]_g$ for any g.

2.3 Semantics for imperatives

In the previous sections we have defined what it means for an imperative to be *executed* in a conduct and what the ways in which an imperative may be realized are.

But what is the effect of the utterance of an imperative? When a speaker utters an imperative such as (13), what do the addressees learn? Certainly, not anything about their *actual* conduct. Rather, they learn something about the conduct the speaker wants them to keep; that is, they learn something about the desire state of the speaker.

We can model a desire state as a set of conducts, which we conceive of as the conducts required/desired/accepted by the speaker.

Definition 2.7 (Desire states)**.** A *desire state*, sometimes called simply a *state*, is a set of conducts.

Here a remark is in place: what an imperative refers to need not *always* be the speaker's desire state or even any actual desire state. For instance, consider the imperative in (16).

(16) To recharge your mobile, visit our website.

In this case the imperative has nothing to do with any agent's desires. However, it still provides information about a *state* in the formal sense, i.e. about a certain set of conducts; what set this is is specified by the initial clause: the set of conducts that would lead the addressees to recharge their mobiles. In general, one may regard an imperative 'in order to do p, do q' as stating that the information that the imperative 'do q' provides is to be interpreted as regarding the set of conducts that lead to p.

At any rate, the task of modeling the designation of the particular desire state at stake lies out of the scope of the present paper. What matters for our purposes is that although what the state in question represents may differ for imperatives like (13) and (16), this difference does not affect the way the imperative works, the information it provides. We can thus safely think of the state in question as the desire state of the speaker, without being in any way committed to this assumption.

Now, what information does an imperative φ provide about this desire state s? First, it provides the information that the speaker wants φ to be executed: hence, that all conducts $M \in s$ are such that $M \models \varphi$, or in other words that $s \subseteq |\varphi|$.

Moreover, if φ is a choice imperative, it grants the permission to freely choose one of the possibilities for φ and to execute that possibility and *only* that. To capture this intuition we use the notion of exclusive strengthening from Roelofsen and van Gool [2010].

Definition 2.8 (Exclusive Strengthening)**.** If S is a set of states and $s \in S$, the exclusive strengthening of s relative to S is the state

$$\text{exc}(s, S) = s - \bigcup\{t \in S \mid s \not\subseteq t\}$$

The exclusive strengthening of the set S is then the set

$$\text{exc}S = \{\text{exc}(s, S) \mid s \in S\}$$

The effect of the exclusive strengthening operation is to turn any possibility into the corresponding 'only that' possibility. Now that we have a formal notion to say 'only' we can clarify what the freedom of choice granted by an imperative amounts to: the imperative φ informs that any element of $\text{exc}[\![\varphi]\!]$ is consistent (i.e. has non-empty intersection) with the given state s. This leads to the following definition.

Definition 2.9 (Semantics for imperatives)**.** Relative to an assignment g, we say that a state s *justifies* an imperative φ, and we write $s, g \Vdash \varphi$, in case the following two conditions are satisfied:

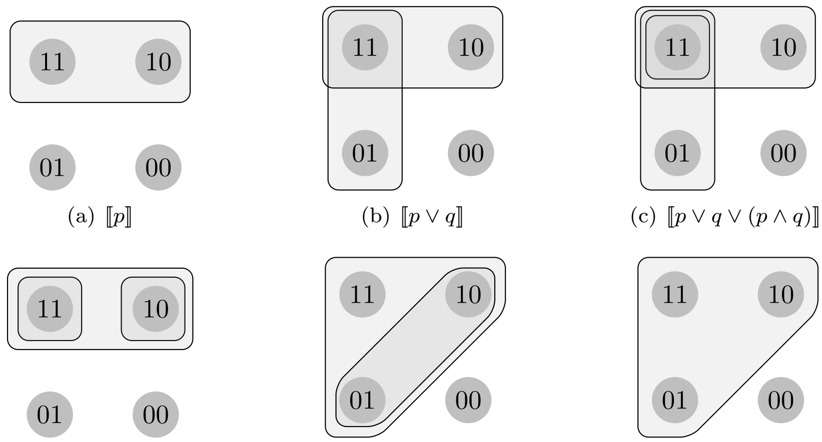
Obligation $s \subseteq |\varphi|_g$

Permission $s \cap t \neq \emptyset$ for all $t \in \text{exc}[\![\varphi]\!]_g$

The *justification set* of φ relative to the assignment g is the set $\langle\varphi\rangle_g$ of states s such that $s, g \Vdash \varphi$.

Of course, this notion can be presented in a dynamic fashion as follows. Upon hearing an imperative φ, an agent learns that the desire state at stake is one that justifies φ, i.e. lies in $\langle\varphi\rangle$.

This learning process is modeled by saying that if an agent knows that the actual desire state lies in a certain set σ, after the utterance of φ the agent shrinks down his options to $\sigma[\varphi] = \sigma \cap \langle\varphi\rangle$.



(a) $[\![p]\!]$ (b) $[\![p \vee q]\!]$ (c) $[\![p \vee q \vee (p \wedge q)]\!]$

(d) $\langle p \rangle$ (e) $\langle p \vee q \rangle$ (f) $\langle p \vee q \vee (p \wedge q) \rangle$

Figure 1: Possibilities and justification sets

2.4 Examples

In this section we will see our system in action in some basic, simple cases. To illustrate our points, we will often take our language to consist only of the propositional letters (nullary predicate symbols) p and q. There are only four models (i.e. valuations) for this language, which we can denote 11, 10, 01 and 00, where 10 is the valuation making p true and q false, and so on.

Example 2.10 (Basic imperatives)**.** As a first example, consider any basic closed imperative φ. Since $[\![\varphi]\!] = \{|\varphi|\}$, also $\mathrm{exc}[\![\varphi]\!] = \{|\varphi|\}$. Now, what states justify φ? First of all, a state justifying φ must be a subset of $|\varphi|$; moreover, it must intersect any element of $\mathrm{exc}[\![\varphi]\!]$, i.e. it must intersect $|\varphi|$. But obviously, any non-empty subset of $|\varphi|$ intersects $|\varphi|$. Hence, $s \Vdash \varphi$ holds iff s is a non-empty subset of $|\varphi|$. For instance, the justification set of the basic imperative p is illustrated in figure 1(d).

Example 2.11 (Disjunction)**.** Let us now consider the simplest choice imperative, namely $p \vee q$. This is justified in a state s in case: first, $p \vee q$ is executed in all conducts in s; and second, there is one conduct in s in which p is executed but not q, and another in which q is executed but not p.

It is easily verified that there are only two states in the language $\mathcal{L} = \{p, q\}$ which justify the utterance of the imperative $p \vee q$, namely $s_0 = \{10, 01\}$ and $s_1 = \{10, 01, 11\}$. Thus, upon hearing $p \vee q$, an agent may conclude that the desire state of the speaker is either s_0 or s_1. This situation is depicted in figure 1(e).

From the imperative $p \vee q$ one learns that doing only p or only q is certainly fine, and that doing neither p nor q is certainly *not* fine, while it remains undetermined whether it is fine to do both p *and* q. We believe this is rightly so, since indeed it seems that an imperative $p \vee q$ does not allow to draw conclusions about the acceptability of the 'doing both' conduct, as witnessed by the fact that the question (17-b) does not sound redundant after (17-a).

(17) a. Go on, take a brownie or a slice of cake!
b. May I have both?

Before turning to the next example, recall that the flattening operation (i.e. double negation) in front of an imperative collapses all possibilities into a single one, thus disabling the choice component and turning the imperative into a basic one.

Example 2.12 (Existential: 'any' and 'some')**.** The behaviour of the imperative $\exists x P(x)$ is analogous to that of a big disjunction over the elements $d \in D$. $\exists x P(x)$ is justified in a state s in case $\exists x P(x)$ is executed in all conducts in s and moreover for any $d \in D$ there is one conduct in s in which $P(d)$ is executed and for no $d' \neq d$ is $P(d')$ executed.

Thus, from the imperative $\exists x P(x)$ one learns that they have to execute $P(d)$ for at least one individual d, and that for any d, executing only $P(d)$ (where *only* means not executing $P(d')$ for any $d' \neq d$) is fine. Thus, existential quantifier models the free-choice item *any*.

On the other hand, consider the flattening $F\exists x P(x)$: according to what we saw, this is a basic imperative which merely conveys the obligation to execute $P(d)$ for some d, without granting permissions. Therefore, the flattened existential models the natural language indefinite *some*.

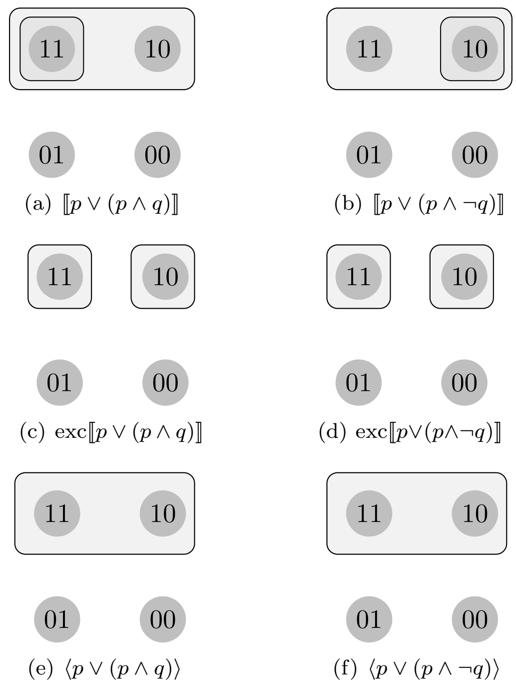
Example 2.13. Now let us consider the imperative $p \vee q \vee (p \wedge q)$, whose possibilities are depicted in figure 1(c). The peculiar feature of this imperative with respect to the previous one lies in the fact that one possibility for this formula is included in other possibilities.

In natural language the imperative "Call Mark, or Andrew, or both" is interpreted as "Call only Mark, or call only Andrew, or call both", i.e. as specifying the obligation to call at least one of Mark and Andrew and together granting the permission to call Andrew and not Mark, to call Mark and not Andrew, and to call both.

Now, we defined the exclusive strengthening of a state s relative to a set of states S by eliminating from s the indices that were already in other possibilities, *but* only in those possibilities which do not include s itself. This restriction was designed precisely to deal with imperatives like $p \vee q \vee (p \wedge q)$ which specify non-maximal possibilities, and which would otherwise turn out to be contradictions.

Instead, with our definitions we obtain the expected prediction $\mathrm{exc}[\![p \vee q \vee (p \wedge q)]\!] = \{|p \wedge \neg q|, |\neg p \wedge q|, |p \wedge q|\} = \mathrm{exc}[\![(p \wedge \neg q) \vee (\neg p \wedge q) \vee (p \wedge q)]\!]$. Thus, $p \vee q \vee (p \wedge q)$ is justified on a state s in case $s \subseteq |p \vee q|$ and moreover s has non-empty intersection with $|p \wedge \neg q|$, with $|q \wedge \neg p|$ and with $|p \wedge q|$. It is easily verified that the only state which satisfies these conditions is $s_0 = \{11, 01, 10\}$, whence a hearer of $p \vee q \vee (p \wedge q)$ learns that the desire state of the speaker is precisely s_0.

To put it in very intuitive terms: $p \vee q \vee (p \wedge q)$ behaves exactly like $p \vee q$, but it also specifies that doing both p and q is fine, whereas this is left undetermined by $p \vee q$.



(a) $[\![p \vee (p \wedge q)]\!]$ (b) $[\![p \vee (p \wedge \neg q)]\!]$

(c) $\mathrm{exc}[\![p \vee (p \wedge q)]\!]$ (d) $\mathrm{exc}[\![p \vee (p \wedge \neg q)]\!]$

(e) $\langle p \vee (p \wedge q) \rangle$ (f) $\langle p \vee (p \wedge \neg q) \rangle$

Figure 2: A problem

Aside: Exclusive strengthening versus exhaustification Consider now the following disjuctive imperative (a simplified version of example (11) from the introduction):

(18) To pass the seminar, write a paper or give a presentation.

Suppose one gets credits for writing a paper (obligatory) with the option to combine it with an oral presentation. In this situation the disjunctive imperative (18) is not justified according to our semantics: for, the state at stake is not consistent with giving a presentation *without* writing a paper. Formally, the given scenario is modeled by the state $s = \{11, 10\}$, the imperative (18) by the formula $p \vee q$, and we see that $s \not\models p \vee q$ since $\{01\} \in \mathrm{exc}[\![p \vee q]\!]$ but $s \cap \{01\} = \emptyset$. This seems to capture precisely the reason why un utterance of (18) would be perceived as deviant in the described situation.

The canasta example discussed in the introduction is accounted for in a similar fashion. For all these examples it is crucial that to justify an imperative

φ a state s must be consistent with any possibility in the *exclusively strengthened* value of the sentence $\text{exc}[\![\varphi]\!]_g$, rather than in its plain set of possibilities $[\![\varphi]\!]_g$. There are, however, some potential problems with this approach. As an illustration, consider the following two imperatives:

(19) a. Call Mark, or call both Mark and Andrew.
     b. Call Mark, or call Mark but not Andrew.

Although the sets of possibilities of these two imperatives are different, their exclusively strengthened values turn out to be the same. Therefore, (19-a) and (19-b) are predicted to be equivalent: both grant the permission to call only Mark or to call both Mark and Andrew (see Figure 2).

A possible solution to this problem would be to substitute the notion of exclusive strengthening with a more general, context dependent notion of *exhaustification*. When told that "Mark and Andrew called" people normally conclude that nobody else called. In the linguistic literature this is called an exhaustive interpretation of the sentence. Exhaustification is a context dependent notion. Which possibilities are excluded depends on the set of relevant alternatives. If the possibility that Mary called is not relevant in the context, from "Mark and Andrew called" we are not entitled to conclude that Mary did not call.

In the given definition of exclusive strengthening, conducts were excluded with respect to the possibilities of the imperative. In the following more general notion of exhaustification, conducts are excluded with respect to the set S of relevant possibilities given by the context.

Definition 2.14 (Exhaustification)**.** $exh(\varphi, S) = \{\text{exc}(s, S) \mid s \in [\![\varphi]\!]\}$

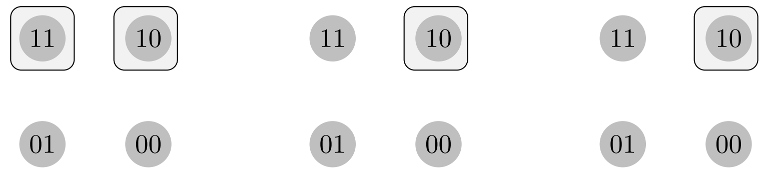
Let us model (19-a) as $p \vee (p \wedge q)$ and (19-b) as $p \vee (p \wedge \neg q)$. In a context in which you wonder whether you should call Mark or Andrew, the set of relevant possibilities is $S = \{p, q\}$. With respect to this set of relevant possibilities, the exhaustified values of the two imperatives are as in Figure 3. By relativizing the semantics of the imperative to exh rather than exc, we would then correctly predict that only (19-a) grants the permission to call both Mark and Andrew. Example (19-b) is predicted to be pragmatically anomalous in that it conveys the same message as p ("Call Mark!"), but in a less perspicuous way. In the remaing of the article we will however ignore exhaustivity and adopt the context independent notion of exclusive strengthening.

2.5 Entailment

We can define entailment between imperatives in the natural way.

Definition 2.15 (Entailment, equivalence)**.** We say that an imperative φ *entails* an imperative ψ, and write $\varphi \models \psi$ in case ψ is justified whenever φ is, that is, in case for any state s and assignment g, $s, g \Vdash \varphi$ implies $s, g \Vdash \psi$.

We say that φ and ψ are *equivalent*, in symbols $\varphi \equiv \psi$, in case $\varphi \models \psi$ and $\psi \models \varphi$, i.e. in case for any state s and assignment g, $s, g \Vdash \varphi \iff s, g \Vdash \psi$.



(a) $exh(p \vee (p \wedge q), \{p, q\})$ (b) $exh(p \vee (p \wedge \neg q), \{p, q\})$ (c) $exh(p, \{p, q\})$

Figure 3: Exhaustive values

As expected, $\varphi \models \psi$ holds iff for any set σ of states we have $\sigma[\varphi][\psi] = \sigma[\varphi]$ (in the update notation defined in section 2.3); this latter characterization makes it clear that φ entails ψ in case φ provides at least as much information as ψ, and possibly more. Spelling out the definition we obtain the following reformulation.

Proposition 2.16. An imperative φ entails an imperative ψ iff the following two conditions hold:

1. entailment of obligations: $|\varphi|_g \subseteq |\psi|_g$ for all g (i.e., φ entails ψ in classical logic);
2. entailment of permissions: for all g, for any $t \in \text{exc}[\![\psi]\!]_g$ there is $s \in \text{exc}[\![\varphi]\!]_g$ with $s \subseteq t$.

From this reformulation it is easy to derive the following corollary.

Corollary 2.17. Two imperatives φ and ψ are equivalent precisely in case $|\varphi|_g = |\psi|_g$ and $\text{exc}[\![\varphi]\!]_g = \text{exc}[\![\psi]\!]_g$ for any assignment g.

Contradictory imperatives are defined in the natural way as imperatives which are never justified.

Definition 2.18 (Contradictions)**.** We say that an imperative φ is a *contradiction* in case $\langle\varphi\rangle_g = \emptyset$ for all g. If φ is not a contradiction we say it is *consistent*.

Remarks

1. Entailment between basic imperatives is utterly classical. For instance, $\forall x P(x) \models P(a)$.
2. For any consistent imperative φ, $\varphi \equiv F\varphi \iff \varphi$ is basic.
3. We avoid the traditional problem in the theory of imperatives, namely Ross' paradox. The imperative "call Andrew" does not entail "call Andrew or Mark" for the very intuitive reason that the former imperative is justified also (and indeed only) in states in which calling Mark instead of Andrew is *not* an option.

4. Any formula φ entails its flattening $F\varphi$, since the obligation specified by $F\varphi$ is already part of the meaning of φ. In particular, $\exists x\varphi \models F\exists x\varphi$ for any φ, that is, 'any' entails 'some'.

5. Viceversa, not granting any permission, a basic imperative can never entail a choice imperative. In particular, in general we have:

   - 'and' does not entail 'or': $p \wedge q \not\models p \vee q$;
   - 'every' does not entail 'any': $\forall x P(x) \not\models \exists x P(x)$;
   - 'some' does not entail 'any': $F\exists x P(x) \not\models \exists x P(x)$.

6. $p \vee q \vee (p \wedge q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q) \vee (p \wedge q) \models p \vee q$.

7. $p \vee (p \wedge q) \equiv (p \wedge \neg q) \vee (p \wedge q) \equiv p \wedge (q \vee \neg q)$.

3 Deontics

3.1 Syntax and semantics of deontics

We have seen how imperatives can be used to provide information about desire states, i.e. sets of conducts. Of course, there is also another language which stands as a natural candidate to talk about such objects, namely a predicate modal language in which any atom lies in the scope of exactly one modality. In this section we will consider this language and relate it with imperatives on the one hand, and with the deontics 'may' and 'must' on the other.

Definition 3.1 (Deontics)**.** The set of deontics is defined as the smallest set such that:

1. if φ is an imperative, $\Diamond\varphi$ is a deontic;

2. if α and β are deontics, so are $\neg\alpha$, $\alpha \vee \beta$, $\alpha \wedge \beta$, $\exists x\alpha$ and $\forall x\alpha$.

As customary in modal logic, we shall use $\Box\varphi$ as a shorthand for $\neg\Diamond\neg\varphi$. This language will be interpreted in the natural way: $\Diamond\varphi$ will be judged justified in a state if φ is consistent with s, and connectives and quantifiers will behave classically.

Definition 3.2 (Semantics for deontics)**.** Let s be a state and let g be an assignment. The justification relation $\Vdash$ is defined as follows:

1. $s, g \Vdash \Diamond\varphi$ in case $s \cap |\varphi|_g \neq \emptyset$;

2. the inductive clauses defining $s, g \Vdash \alpha$ for complex αs are the classical ones.

Entailment and equivalence are defined like for imperatives, and it is also perfectly meaningful to talk of an entailment or an equivalence between a deontic and an imperative. However, in order to make things run smooth in this comparison –which will be the subject of the next section– we should restrict our semantics to non-empty states.

Indeed, the empty state represents the inconsistent desire state, in which no conduct is considered acceptable. It seems reasonable to think that when interpreting an imperative, one assumes the relevant desire state to be consistent -otherwise what are we talking about?

This came by itself for imperatives, since according to our definitions no imperative is justified on the empty state anyway. However, since deontics of the shape $\Box\varphi$ are justified on $\emptyset$, we should now be explicit. This minor modification is also needed to make definition 3.5 work well for imperatives.

3.2 Imperatives as deontics

Recall from section 2.2 that if the domain D is finite we have a finite, well-defined set $\mathcal{R}(\varphi)$ of formulas called realizations which express the possibilities for φ. In this section we will see that in this case, imperatives are a particular class of deontics, in the sense that they can be translated into deontics with the same semantic content. Notice that this case covers all propositional imperatives.

The first step is to specify how the semantic operation of exclusive strengthening may be reproduced at the syntactic level.

Definition 3.3 (Exclusive Strengthening)**.** If Φ is a finite set of formulas and $\varphi \in \Phi$, we put

1. $\mathrm{exc}(\varphi, \Phi) = \varphi \wedge \bigwedge\{\neg\psi \,|\, \psi \in \Phi$ and φ does not classically entail $\psi\}$
2. $\mathrm{exc}\Phi = \{\mathrm{exc}(\varphi, \Phi) \,|\, \varphi \in \Phi\}$

As expected, we have $\mathrm{exc}[\![\varphi]\!]_g = \{|\rho|_g \,|\, \rho \in \mathrm{exc}\mathcal{R}(\varphi)\}$.

Proposition 3.4. If the domain D is finite, then for any formula φ,

$$\varphi \equiv \Box\varphi \wedge \bigwedge\{\Diamond\rho \,|\, \rho \in \mathrm{exc}\mathcal{R}(\varphi)\}$$

This representation makes it even clearer that the meaning of an imperative φ consists of two components: an obligation $\Box\varphi$ and a permission $\bigwedge\{\Diamond\rho \,|\, \rho \in \mathrm{exc}\mathcal{R}(\varphi)\}$. Observe that the obligation component of an imperative φ coincides with its flattening: $\Box\varphi \equiv F\varphi$.

As an example, we have the following equivalence, in which the decomposition into obligation and permission is made explicit for a disjunctive imperative.

(20) $\quad \varphi \vee \psi \equiv \Box(\varphi \vee \psi) \wedge \Diamond(\varphi \wedge \neg\psi) \wedge \Diamond(\psi \wedge \neg\varphi)$

In some cases (but not in general) we may find a deontic formula expressing an imperative independently of the size of the domain. For instance, the quantifier analogue of the equivalence (20) holds in general[2]:

[2]Strictly speaking, here we are cheating. For, our language does not include the equality predicate, and thus we cannot formulate the expression $\forall y \neq x(\neg Py)$. However, this problem can be easily overcome, for instance by introducing directly quantifiers $\exists x \neq y, \forall x \neq y$ in the syntax of imperatives. The associated semantic clauses are straightforwardly formulated.

(21) $\exists x P(x) \equiv \Box \exists x P(x) \wedge \forall x \Diamond (Px \wedge \forall y \neq x(\neg Py))$

Now the time has come to be more precise about what it means for a deontic to be an obligation or a permission. Intuitively, obligations carry the information that the desire state s in question is *at least as narrow* (recall that the narrower a state, the more demanding!) as to entail that certain actions must be executed, while permissions carry the information that s is *at least as large* as to be consistent with the execution of certain actions. Thus, obligations are downward persistent, while permissions are upwards persistent. We can take these persistency properties as definition of obligations and permissions.

Definition 3.5 (Obligations and permissions)**.**

- We say that a deontic α is an *obligation* if it is downward persistent, i.e. if for all states $s \subseteq t$ and assignments g, if $t, g \Vdash \alpha$ also $s, g \Vdash \alpha$.
- We say that a deontic α is a *permission* if it is upward persistent, i.e. if for all states $s \subseteq t$ and assignments g, if $s, g \Vdash \alpha$ also $t, g \Vdash \alpha$.

Examples of permissions and obligations are easily given: deontics of the shape $\Diamond\varphi$ are always permissions, and deontics of the shape $\Box\varphi$ are always obligations. Moreover, the next proposition states that both classes are closed under the positive logical constants ($\vee, \wedge, \exists$ and $\forall$) while negation turns obligations into permissions and viceversa.

Proposition 3.6. 1. If α and β are obligations (resp. permissions), then so are $\alpha \vee \beta$, $\alpha \wedge \beta$, $\exists x\alpha$ and $\forall x\alpha$.

2. If α is an obligation (resp. permission) then $\neg\alpha$ is a permission (resp. obligation).

Obviously, the notions of permission and obligation also apply to imperatives. Quite naturally, a basic imperative is always an obligation; in fact, something slightly stronger can be said.

Remark 3.7. A consistent imperative is an obligation iff it is basic.

Thus, also a consistent imperative φ is an obligation iff $\varphi \equiv F\varphi \equiv \Box\varphi$. On the other hand, there are imperatives which are permissions.

Remark 3.8. All classical tautologies are permissions. That is, any imperative φ such that $|\varphi|_g = \mathcal{I}_D$ for all g is a permission.

That imperatives may be used to grant permissions may strike as odd, but as a matter of fact an imperative like "Call Mark or don't call him" (usually accompanied by "do as you wish" or similar) is not a triviality: it provides the information that both options are acceptable, which is exactly what we predict for $p \vee \neg p$.

3.3 'May' and 'must'

We have seen that an imperative φ can be decomposed as the conjunction of two deontics, an obligation $\Box\varphi \equiv F\varphi$ and a permission $\bigwedge\{\Diamond\rho \mid \rho \in \mathrm{exc}\mathcal{R}(\varphi)\}$. One hypothesis that we could make is that these two deontics mirror the natural language expressions 'you must do φ' and 'you may do φ'. The idea is to model 'must' as just a plain box, as 'may' as the operator

$$\textsc{may}(\varphi) := \bigwedge\{\Diamond\rho \mid \rho \in \mathrm{exc}\mathcal{R}(\varphi)\}$$

The problem with this is that the resulting MAY is defined only under the annoying restriction of finite domains. But nothing prevents us from taking MAY as a primitive operator, with its own semantics.

Definition 3.9 (Semantics of *may*)**.**
$s \Vdash_g \textsc{may}(\varphi) \iff s \cap t \neq \emptyset$ for all $t \in \mathrm{exc}[\![\varphi]\!]_g$

Thus, for instance, $\textsc{may}(p)$ simply provides the information that the relevant state is consistent with doing p; $\textsc{may}(p \vee q)$ provides the information that it is consistent with doing only p and only q; and $\textsc{may}(\exists x Px)$ provides the information that it is consistent with choosing an element $d \in D$ and doing Px only for $x = d$. Formally, we have the following equivalences.

(22) a. $\textsc{may}(p) \equiv \Diamond p$
b. $\textsc{may}(p \vee q) \equiv \Diamond(p \wedge \neg q) \wedge \Diamond(q \wedge \neg p)$
c. $\textsc{may}(\exists x Px) \equiv \forall x \Diamond(Px \wedge \forall y \neq x(\neg Py))$

With the operator MAY in place, the representation of an imperative as a conjunction of an obligation and a permission becomes a general fact.

Proposition 3.10. For any imperative φ,

- $\varphi \equiv \Box\varphi \wedge \textsc{may}(\varphi)$;
- $\Box\varphi$ is an obligation;
- $\textsc{may}(\varphi)$ is a permission.

This account of *must* and *may* has the merit of predicting the free choice nature of permissions in contrast to the 'flatness' of obligations and the difference between 'may' and 'must' with respect to their licensing of free choice *any* (along the lines of Aloni [2007]).

(23) a. You may call John or Mary $\Rightarrow$ You may call (only) John and you may call (only) Mary.
b. You must call John or Mary $\not\Rightarrow$ You may call (only) John and you may call (only) Mary.

(24) a. You may call anyone.
b. #You must call anyone.

4 Conclusion

We have presented a semantics for imperatives in the framework of inquisitive semantics [Groenendijk and Roelofsen, 2009, Ciardelli, 2009]. Basic free choice inferences follow as entailments. Ross' paradox in (1) is explained. The contrast in grammaticality illustrated in (3) follows along the lines of Aloni [2007]. Negative imperatives are never choice offering on this account, while universal ones might be (*contra* Aloni [2007]). Via the notion of exclusive strengthening we further improved on Aloni's predictions with respect to examples (11) and (12).

5 Further work

The aim of this article was to propose the core of a semantics for imperatives. Several natural lines of research stemming from this bulb can be envisaged.

First of all, we said at the beginning that in this context formulas denote actions, not propositions. However, we then stuck to a basic modeling of actions by means of first-order models. It would be interesting to investigate more refined approaches, in particular in order to incorporate sequentiality.

A related problem is that of dealing with the interaction between imperatives and indicatives, such as in conditional imperatives of the form "if μ then φ", where μ is an indicative and φ is an imperative.

Also, we remarked in section 2.3 that the desire state which is relevant for the interpretation of an imperative may be determined by means of expressions like "in order to ..." and similar, see example (16). However, our simple model did not include any mechanism to account for such constructions.

The logic associated to this system remains to be investigated, as well as the empirical value of the account of deontic *may* and *must* suggested in section 3.3.

Finally, our semantics does not explain why, unlike deontics, imperatives only admit a performative reading and lack a descriptive one (see Schwager [2006] for a possible explanation).

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