Knowing-who in quantified epistemic logic

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Abstract This article proposes an account of knowing-who constructions within a generalisation of Hintikka's (1962) quantified epistemic logic employing the notion of a conceptual cover (Aloni, 2001). The proposed logical system captures the inherent context-sensitivity of knowing-*wh* constructions (Boër and Lycan, 1985), as well as expresses non-trivial cases of so-called concealed questions (Heim, 1979). Assuming that quantifying into epistemic contexts and knowing-who are linked in the way Hintikka had proposed, the context dependence of the latter will translate into a context dependence of *de re* attitude ascriptions and this will result in a ready account of a number of traditionally problematic cases including Quine's well-known double vision puzzles (Quine, 1956).

Keywords: quantified epistemic logic, knowing-who constructions, concealed questions, propositional attitudes, context-dependence

1 Introduction

Hintikka (1962) famously developed the first systematic formal study of the notions of knowledge and belief. His epistemic modal logic provided new insights into the properties of epistemic agents and their attitude about the world, its objects and its states of affairs. One of the most controversial issues in the quantified version of Hintikka's epistemic logic (QEL) concerns the possibility of existential generalisation (**EG**) from an epistemic context (e.g. Frege, 1892; Quine, 1956; Kaplan, 1969; Carlson, 1988; Holliday and Perry, 2014). QEL invalidates unrestricted versions of **EG** because from $\Box \psi[t]$ we cannot always infer $\exists x \Box \psi[x]$, where $\Box$ stands for an arbitrary epistemic necessity operator:

EG $\not\models_{QEL} \phi[t] \rightarrow \exists x \phi[x]$

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In order for existential generalisation to be applicable to a term t occurring in the scope of an epistemic modal, t has to denote the same individual in all epistemic alternatives of the relevant agent. The latter condition can be expressed by the formula $\exists x \Box x = t$ in QEL. The following principle is QEL-valid, if we assume consistency, positive introspection, and negative introspection:[1]

$$\mathbf{EG}_\Box \quad \models_{QEL} \exists x \Box x = t \rightarrow (\Box \phi[t] \rightarrow \exists x \Box \phi[x])$$

Formulas of the form $\exists x \Box x = t$ are used by Hintikka as representations of knowing-who constructions. $\mathbf{EG}_\Box$ says that one can existentially generalise from a term t in an epistemic context if we have as an additional premise that the relevant agent knows who t is. Formulas of the form $\exists x \Box \phi[x]$ are used to express *de re* attitude reports like "There is someone whom Ralph believes/knows to be a spy". In Hintikka's epistemic logic, in order to have a *de re* attitude about a person one needs to know who the person is.

The evaluation of knowing-who constructions in ordinary language, however, is a complex matter, as Hintikka himself acknowledged. In *Knowledge and Belief* he wrote:

> In practice it is frequently difficult to tell whether a given sentence of the form 'a knows who b is' or $\exists x K_a(b = x)$ is true or not. The criteria as to when one may be said to know who this or that man is are highly variable (Knowledge and Belief, p. 149n).

The goal of the present article is to present an explicit account of this variability within a generalisation of Hintikka's epistemic logic employing the notion of a conceptual cover (Aloni, 2001, 2005b). The resulting logical system will allow us to capture the inherent context-sensitivity of knowing-*wh* constructions (Boër and Lycan, 1985), as well as to express non-trivial cases of so-called concealed questions (Heim, 1979). Assuming that quantifying into epistemic contexts and knowing-who are linked in the way Hintikka had proposed, the context dependence of the latter will translate into a context dependence of *de re* attitude ascriptions and this will result in a ready account of a number of traditionally problematic cases including Quine's well-known double vision puzzles (Quine, 1956).

2 On the variability of knowing-*wh*

Imagine the following situation from Aloni (2001). In front of you lie two face-down cards. One is the Ace of Hearts, the other is the Ace of Spades, but you don't know which is which. You have to choose one card: if you choose the Ace of Hearts you win \$10, if you choose the Ace of Spades you lose \$10. Now consider the following sentence:

(1)   You know which card is the winning card.

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[1] If we consider also non-serial, non-transitive and non-euclidean frames, the principle is valid only if ϕ does not contain any modal operator.

Is this sentence true or false in the given situation? On the one hand, the sentence is true: you know that the Ace of Hearts is the winning card. If someone interested in the rules of the game asked you "Which card is the winning card?", you would be able to answer in an appropriate way. On the other hand, suppose someone interested in winning the game would ask you "Which card is the winning card?" In this case you would not be able to answer in the desired way: as far as you know, the winning card may be the card on the left, but it may just as well be the card on the right. Therefore you don't know which card is the winning card (similar "yes and no" cases were discussed in Boër and Lycan, 1985).

Aloni (2001) proposed the following explanation of this example. Intuitively, there are two ways in which the cards may be identified in this situation: by their position (the card on the left, the card on the right) or by their suit (the Ace of Hearts, the Ace of Spades). Whether (1) is judged true or false seems to depend on which of these perspectives is adopted. If identification by suit is adopted, as in the first context discussed above, the sentence is judged true. But if identification by position is adopted, as in the second context, the sentence is judged false.

Aloni (2001, 2005b) proposed to formalise identification methods by means of *conceptual covers*. A conceptual cover is a set of individual concepts (functions from possible worlds to individuals) that satisfies the following condition: in a conceptual cover, in each world, each individual constitutes the value (or instantiation) of one and only one concept.

Definition 1 (Conceptual cover). Given a set of possible worlds W and a universe of individuals D, a *conceptual cover CC* based on (W, D) is a set of functions $W \to D$ such that:

$$\forall w \in W : \forall d \in D : \exists! c \in CC : c(w) = d.$$

Conceptual covers are sets of concepts which exhaustively and exclusively cover the domain of individuals. In a conceptual cover each individual d is identified by at least one concept in each world (*existence*), but in no world is an individual counted more than once (*uniqueness*).

It is easy to prove that each conceptual cover and the domain of individuals have the same cardinality. In a conceptual cover, each individual is identified by one and only one concept. Different covers constitute different ways of conceiving one and the same domain.

For the sake of illustration consider again the card scenario described above. In that scenario there are at least three salient ways of identifying the cards which can be represented by the following conceptual covers: (2-a) represents identification by ostension, (2-b) represents identification by name, and (2-c) represents identification by description (cf. Hintikka, 1972).

(2)   a.   {on-the-left, on-the-right}   [ostension]
      b.   {ace-of-spades, ace-of-hearts}   [naming]
      c.   {the-winning-card, the-losing-card}   [description]

The set of concepts in (3) is not an example of a conceptual cover because it does not satisfy the conditions formulated in our definition.

(3)   #{on-the-left, ace-of-spades}

Intuitively, (3) does not represent a proper perspective over the relevant domain of individuals: as far as we know, the card on the left might be the Ace of Spades. If so: (i) one card (the Ace of Spades) would be counted twice; and (ii) another card (the Ace of Hearts) would not be identified at all.

When we talk about concepts, we implicitly assume two different levels of 'objects': the individuals (in D) and the ways of referring to these individuals (in D^W). An essential feature of the intuitive relation between the two levels of the individuals and of their representations is that to one element of the first set correspond many elements of the second: one individual can be identified in many different ways. What characterises a set of representations of a certain domain is this cardinality mismatch, which expresses the possibility of considering an individual under different perspectives which may coincide in one world and not in another. Individuals, on the other hand, do not split or merge once we move from one world to the other. Now, since the elements of a cover also cannot merge or split (by uniqueness), they behave like individuals in this sense, rather than representations. On the other hand, a cover is not barely a set of individuals, but encodes information on how these individuals are specified. We thus can think of covers as sets of individuals each identified in one specific way. My proposal is that knowing-*wh* constructions involve quantification over precisely this kind of sets. By allowing different conceptual covers to constitute the domain of quantification on different occasions, we can account for the "yes and no" cases discussed above, without failing to account for the intuition that knowing-*wh* constructions involve quantification over genuine individuals, rather than over ways of specifying these individuals.

In the semantics for quantified epistemic logic presented in the next section the evaluation of formulas is relativised to a contextual parameter which assigns conceptual covers to variables as their domain of quantification. Building on (Hintikka, 1972), formula (5) will be used as the logical representation of (4).[2] The variable z_n in (5) is indexed by a CC-index $n \in N$ ranging over conceptual covers. The evaluation of (5) will vary relative to the contextually selected value of n as illustrated in (6).

(4)   You know which card is the winning card.

(5) $\exists z_n \Box z_n = c$

(6)   a.   False, if $n \mapsto \{$on-the-left, on-the-right$\}$
        b.   True, if $n \mapsto \{$ace-of-spades, ace-of-hearts$\}$
        c.   Trivial, if $n \mapsto \{$the-winning-card, the-losing-card$\}$

Differently indexed variables z_n and z_m will be allowed to range over different conceptual covers allowing perspicuous representations of traditionally problematic cases including Quine's double vision puzzles.

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[2] Definite descriptions will be translated as (non-rigid) individual constants until section 4.2.

3 QEL under conceptual covers

3.1 Language

We assume a set $\mathscr{C}$ of individual constants, a set $\mathscr{P}$ of predicates, and an enumerable set $\mathscr{V}_N$ of CC-indexed individual variables. Then we define the terms t and formulas ϕ of our *language* $\mathscr{L}_{CC}$ by the following BFN:

$$t := c \mid x_n \tag{1}$$

$$\phi := Pt_1,\ldots,t_m \mid t_1 = t_2 \mid \neg\phi \mid \phi \wedge \phi \mid \exists x_n \phi \mid \Box\phi \tag{2}$$

where $c \in \mathscr{C}$, $x_n \in \mathscr{V}_N$, and $P \in \mathscr{P}$.

The usual abbreviations for $\vee$ ('disjunction'), $\rightarrow$ ('implication'), $\leftrightarrow$ ('bi-implication'), $\forall$ ('universal quantifier') and $\diamond$ ('possibility') apply.

3.2 Semantics

A CC-*model* for $\mathscr{L}_{CC}$ is a quintuple $\langle W, R, D, I, C \rangle$ in which W is a non-empty set of possible worlds; R is a relation on W; D is a non-empty set of individuals; I is an interpretation function which assigns for each $w \in W$ an element $I_w(c)$ of D to each individual constant c in $\mathscr{C}$, and a subset $I_w(P)$ of D^m to each m-ary predicate P in $\mathscr{P}$; and C is a set of conceptual covers over (W, D).

Definition 2 (CC-assignment). Let $K = \{f \cup h \mid f \in C^N \,\&\, h \in D^{W^{\mathscr{V}_N}}\}$. A CC-assignment g is an element of K satisfying the following condition: $\forall n \in N$: $\forall x_n \in \mathscr{V}_N$: $g(x_n) \in g(n)$.

A CC-assignment g has in this system a double role: it works on CC-indices and on indexed variables. CC-indices, n, are mapped to conceptual covers elements of C. n-indexed individual variables, x_n, are mapped to concepts elements of $g(n)$.

Well-formed expressions in $\mathscr{L}$ are interpreted in models with respect to a CC-assignment function g and a world $w \in W$.

Definition 3 (Interpretation of Terms).

$$[\![t]\!]_{M,w,g} = g(t)(w) \text{ if } t \text{ is a variable} \tag{3}$$

$$[\![t]\!]_{M,w,g} = I_w(t) \text{ if } t \text{ is a constant} \tag{4}$$

Definition 4 (Interpretation of Formulas).

$$M,w \models_g Pt_1,\ldots t_n \ \text{ iff } \ \langle [\![t_1]\!]_{M,w,g}, \ldots, [\![t_n]\!]_{M,w,g} \rangle \in I_w(P) \tag{5}$$

$$M,w \models_g t_1 = t_2 \ \text{ iff } \ [\![t_1]\!]_{M,w,g} = [\![t_2]\!]_{M,w,g} \tag{6}$$

$$M, w \models_g \neg\phi \quad \text{iff} \quad \text{not } M, w \models_g \phi \tag{7}$$

$$M, w \models_g \phi \wedge \psi \quad \text{iff} \quad M, w \models_g \phi \text{ and } M, w \models_g \psi \tag{8}$$

$$M, w \models_g \exists x_n \phi \quad \text{iff} \quad \exists c \in g(n) : M, w \models_{g[x_n/c]} \phi \tag{9}$$

$$M, w \models_g \square\phi \quad \text{iff} \quad \forall w' : wRw' : M, w' \models_g \phi \tag{10}$$

In this semantics quantifiers range over elements of contextually determined conceptual covers, rather than over individuals *simpliciter* (clause (9) in definition 4).

Remark 1. An essential feature of this semantics is that differently indexed variables x_n and y_m may range over different sets of concepts. This will be crucial in a number of applications discussed later on, see examples (8)-(10), (21), (26) and (34).

Remark 2. It should be stressed however that the denotation $[\![x_n]\!]_{M,w,g}$ of a variable x_n with respect to a model M, a world w and an assignment function g is not the concept $g(x_n) \in g(n)$, but rather the value $g(x_n)(w)$ of the concept $g(x_n)$ in world w, i.e. an individual in D (see clause (3) in definition 3). Thus, variables do not refer to concepts, but to individuals. However, they do refer in a non-rigid way: different individuals can be their value in different worlds.

All other semantic clauses are defined as in standard quantified modal logic, as is the notion of validity. A formula is valid in a CC-model M iff it is true with respect to all assignments and all worlds in M. A formula is CC-valid iff it is valid in all CC-models.

Definition 5 (CC-Validity). Let $M = \langle W, R, D, I, C \rangle$ be a CC-model for $\mathscr{L}_{CC}$ and ϕ a formula of $\mathscr{L}_{CC}$.

$$M \models \phi \quad \text{iff} \quad \forall w \in W, \forall g : M, w \models_g \phi$$

$$\models_{CC} \phi \quad \text{iff} \quad \forall M : M \models \phi$$

3.3 Axiomatisation

Aloni (2005b) showed that the semantics presented above can be axiomatised by the following set of axiom schemata:[3]

Basic propositional modal system

PC All propositional tautologies.
  K $\square(\phi \rightarrow \psi) \rightarrow (\square\phi \rightarrow \square\psi)$

Quantifiers Recall that $\phi[t]$ and $\phi[t']$ differ only in that the former contains the term t in one or more places where the latter contains t'.

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[3] This axiomatisation taken from Aloni (2005b) is based on the axiom system of modal predicate logic with identity in Hughes and Cresswell (1996). See in particular chapters 13, 14 and 17.

EGa $\phi[t] \rightarrow \exists x_n \phi[x_n]$ (if ϕ is atomic)
EGn $\phi[y_n] \rightarrow \exists x_n \phi[x_n]$
BFn $\forall x_n \Box \phi \rightarrow \Box \forall x_n \phi$

Identity

ID $t = t$
SIa $t = t' \rightarrow (\phi[t] \rightarrow \phi[t'])$ (if ϕ is atomic)
SIn $x_n = y_n \rightarrow (\phi[x_n] \rightarrow \phi[y_n])$
LNIn $x_n \neq y_n \rightarrow \Box \, x_n \neq y_n$

The axioms **EGa** and **SIa** govern existential generalisation and substitutivity of identicals for arbitrary singular terms in atomic formulae (generalisable to all non-modal contexts). **EGn** and **SIn** cover the case for simple variables for general formulae. Note that **EGa** expresses the existence condition on conceptual covers and **SIn** the uniqueness condition.

Let AX_{CC} be the set of axioms of CC. The set of CC-theorems T_{CC} is the smallest set such that:

AX $AX_{CC} \subseteq T_{CC}$
MP If ϕ and $\phi \rightarrow \psi \in T_{CC}$, then $\psi \in T_{CC}$
∃I If $\phi \rightarrow \psi \in T_{CC}$ and x^n not free in ψ, then $(\exists x_n \phi) \rightarrow \psi \in T_{CC}$
N If $\phi \in T_{CC}$, then $\Box \phi \in T_{CC}$

I will use the standard notation and write $\vdash_{CC} \phi$ for $\phi \in T_{CC}$.

Theorem 1 (Soundness and Completeness). $\vdash_{CC} \phi$ *iff* $\models_{CC} \phi$.

The next section discusses how the proposed variant of Hintikka's epistemic logic can be used to capture the context-sensitivity of knowing-*wh* constructions observed by Hintikka and illustrated in section 2. Section 5 will compare CC-validity with classical QEL-validity, and discuss a number of applications to canonical examples of propositional attitude reports.

4 Knowing-who and concealed questions

4.1 Context-sensitivity of knowing-*wh*

Consider again the card situation described in section 2. In front of you lie two face-down cards. One is the Ace of Hearts, the other is the Ace of Spades. You don't know which is which. Furthermore, assume that one of the cards is the winning card, but you don't know which. This situation can be modelled as follows (the dot indicates the winning card):

$$w_1 \mapsto \heartsuit \quad \spadesuit^\bullet$$
$$w_2 \mapsto \spadesuit \quad \heartsuit^\bullet$$
$$w_3 \mapsto \heartsuit^\bullet \quad \spadesuit$$
$$w_4 \mapsto \spadesuit^\bullet \quad \heartsuit$$

Of the conceptual covers definable over such sets of worlds W and individuals D three appear to be salient in the described situation, namely:

$A = \{\lambda w[\text{left}]_w, \lambda w[\text{right}]_w\}$
$B = \{\lambda w[\heartsuit]_w, \lambda w[\spadesuit]_w\}$
$C = \{\lambda w[\text{winning}]_w, \lambda w[\text{losing}]_w\}$

These covers correspond to three different ways of identifying individuals, which are available in such a situation: A identifies the cards by ostension. B identifies the cards by their name. C identifies the cards by description.

Suppose now that you learn that the Ace of Hearts is the winning card, but still you don't know whether it is the card on the left or the one on the right. In this situation your epistemic state corresponds to the set: $\{w_2, w_3\}$. Consider the following sentence:

(7)   a.   You know which card is the winning card.
        b. $\exists z_n \square z_n = c$

On the present approach, sentence (7) obtains different evaluations when interpreted under different conceptual covers. Under an assignment which maps n to cover A, i.e., if the operative conceptual cover is the one which cross-identifies objects by pointing at them, the sentence is false, because there is no unique element of A corresponding to the winning card in both your epistemic alternatives: in w_2, the card on the right is the winning card, in w_3, the card on the left. In contrast, if our assignment maps n to cover B, i.e., if the operative cover is the one which cross-identifies objects by their name, then (7) is true, because we can find a concept in B which corresponds to the winning card in all your epistemic alternatives, namely $\lambda w[\heartsuit]_w$. Finally, if our assignment maps n to cover C, the sentence is again evaluated as true, but then in a trivial way. Assuming that the contextual selection of a conceptual cover is governed by Gricean principles of conversation would rule out such a resolution in the given scenario.

A second application of the present semantics concerns sentences like (8):

(8)   a.   You don't know which card is which.
        b. $\forall x_n \forall y_m \lozenge x_n = y_m$

Since our semantics crucially allows different variables to range over different sets of concepts, (8-b) can be used to represent the total ignorance of the sort expressed by (8-a). These cases were problematic in standard QEL. Formula (8-b) could not serve as a representation of (8-a) there because it would have entailed that your epistemic state is inconsistent (if $|D| > 1$). One way to represent in QEL what (8-a) expresses is perhaps the following:

(9) $\forall x \diamond x = a_1 \wedge \ldots \wedge \forall x \diamond x = a_n$

This representation however cannot be generalised to the infinite case and depends on the availability of individual constants of the relevant kind. The CC-semantics instead offers us a principled way to express (8-a), which does not use constants and generalises to all cases. The following can also be expressed in the present semantics:

(10)   a.   You know which card is which.
        b. $\forall x_n \exists y_m \Box x_n = y_m$

Again since x_n and y_m can range over different conceptual covers, formula (10-b), which in classical quantified modal logic is a logical validity, can here be used to express a contingent fact.[4]

    As a third application, notice that the proposed semantics gives a ready account of the following sort of case (Boër and Lycan 1986):

(11)   Alphonse, you don't know who you are: you are the rightful heir to the Albanian throne.

Again, these kind of examples were problematic for Hintikka's original theory. The standard logical rendering of the first clause in (11) is as follows:

(12)   a.   You don't know who you are.
        b. $\neg \exists x \Box you = x$

Assuming with Kaplan that indexicals like *you* are rigid designators, example (12) was wrongly predicted to require for its truth that Alphonse failed to know a tautology. On the present analysis, instead, the intended meaning of (11) can be easily captured by letting x range over elements of a descriptive cover with the concept 'the rightful heir to the Albanian throne' as one of its elements.

    It is however important to stress again that given the constraints we have put on conceptual covers, quantification under conceptual cover functions logically exactly the same as quantification over individuals (Aloni, 2005b). An element of a conceptual cover stands for an individual specified in a determined way, rather than for a way of specifying an individual.

    The next section provides independent motivation for a conceptual cover analysis looking at the case of concealed questions.

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[4] Notice that (8-b) above is stronger than the negation of (10-b) (if $|D| > 2$). Embedded questions have been observed to exhibit so-called *homogeneity* effects (Križ, 2015): "*a* knows who ϕ" is intuitively true if *a* is fully informed about who ϕ, whereas its negation "*a* doesn't know who ϕ" conveys that *a* has pretty much no idea who ϕ. A compositional account of the semantics of the English examples would have to account for these facts (among others). Such a compositional analysis, however, is outside the scope of this article.

4.2 Concealed questions

A concealed question (henceforth CQ) is a noun phrase naturally read as an identity question. As an illustration, consider the italicised nouns in the following examples:[5]

(13)   a.   John knows *the price of milk*.
        b.   John knows what the price of milk is.

(14)   a.   Mary discovered *the murderer of Smith*.
        b.   Mary discovered who the murderer of Smith is.

(15)   a.   They revealed *the winner of the contest*.
        b.   They revealed who the winner of the contest was.

Concealed questions arise not only with definite determiner phrases, but also with indefinite and quantified ones, as illustrated in (16):

(16)   a.   John knows a doctor who could help you.   (Frana 2006)
        b.   John knows every phone number.   (Heim 1979)

Heim (1979) further discussed structurally more complex cases like (17) and observed that such CQ-containing CQs (CCQs) are ambiguous between two readings, which are generally referred to in the literature as Reading A and Reading B:

(17)   John knows the capital that Fred knows.

        a.   <u>Reading A</u>: There is exactly one country x such that Fred can name x's capital; and John can name x's capital as well.
        b.   <u>Reading B</u>: John knows which country x is such that Fred can name x's capital (although John may be unable to name x's capital himself).

Suppose Fred knows that the capital of Italy is Rome. Then on Reading A, (17) entails that John also knows that the capital of Italy is Rome. On Reading B, (17) lacks this entailment. It only follows that John knows that Fred can name the capital of Italy.

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[5] It is well known that English *know* also allows acquaintance readings:

(i)   Mary knows the capital of Italy.
        a.   Acquaintance reading: Mary is acquainted with Rome.
        b.   Concealed Question reading: Mary knows what the capital of Italy is.

In languages like Italian and Dutch, where epistemic *know* and acquaintance *know* are lexically distinct, the CQ reading can be forced by using the verb for epistemic know (*sapere* in Italian and *weten* in Dutch), as shown by (ii) for Italian, which does not allow acquaintance interpretations.

(ii)   Maria sa   la capitale dell'Italia.
        Maria knows$_{EPI}$ the capital   of-the-Italy
        'Mary knows what the capital of Italy is'

A clarification of the logical forms of these sentences turns out not to be a trivial task. Heim in her seminal article considered three possible logical analyses for the basic examples in (13)-(15) but none of these could be extended to a proper analysis of both the cases illustrated in (16) and (17).

In what follows we show how a unified account of the interpretation of definite, indefinite and quantified CQs and of Heim's CCQ-ambiguity can be given using the epistemic logic introduced in the previous section.

The main idea consists in analysing constructions like "*a* knows α", where α is a concealed question, using Hintikka's logical rendering of the semantically equivalent "*a* knows what/who α is" and then interpreting the resulting formula using the epistemic logic under conceptual covers presented in the previous section:[6]

(18) a. *a* knows α
      b. $\exists y_n \Box y_n = \alpha$

In the following illustrations we make use of multi-agent knowledge operators K_a for agent a, interpreted as the necessity modal operator $\Box$ from above and of the following abbreviation: $t = \iota y_n.\phi[y_n]$ stands for $\exists y_n \forall z_n((\phi[z_n] \leftrightarrow y_n = z_n) \wedge y_n = t)$.

Illustrations

First consider a 'plain' definite CQ. (19-a) can be analysed using Hintikka's logical rendering of the semantically equivalent *John knows what the capital of Italy is*.

(19) a. John knows the capital of Italy.
      b. $\exists x_n K_j x_n = \iota y_n.\text{CAPITAL-OF-ITALY}(y_n)$

Formula (19-b) is then interpreted according to the semantics given in the previous section. The intended reading is obtained if x_n is taken to range over the *naming* cover:

(20) $n \rightarrow \{\text{Berlin, Rome, } \dots\}$

Example (21) illustrates our analysis of quantified CQs:

(21) a. John knows every European capital.
      b. $\forall x_n(\text{EUROPEAN-CAPITAL}(x_n) \rightarrow \exists z_m K_j x_n = z_m)$

The most natural cover resolution for n and m here is the following:

(22) a. $n \rightarrow \{\text{the capital of Germany, the capital of Italy, } \dots\}$
      b. $m \rightarrow \{\text{Berlin, Rome, } \dots\}$

The sentence is then predicted to be true iff for each European country John can name the capital of that country. Notice that contrary to the previous example, the

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[6] Aloni (2008) also uses conceptual covers to account for concealed questions, but there concealed questions are analysed as questions employing Groenendijk and Stokhof's (1984) partition theory.

quantified case crucially requires that x_n and z_m range over two different covers, otherwise (21-b) would be trivially true (i.e., it would denote a tautology in every world, relative to any assignment function).

Finally, we turn to Heim's CCQ ambiguity. As we mentioned above, Heim (1979) observed that sentences like (23) have the two readings paraphrased in (24) and (25).

(23)   John knows the capital that Fred knows.

(24)   Reading A: There is exactly one country x such that Fred can name x's capital; and John can name x's capital as well.

(25)   Reading B: John knows which country x is such that Fred can name x's capital (although John may be unable to name x's capital himself).

On the present account, Heim's ambiguity can be easily represented as a scope ambiguity (CC-indices are indicated only on the first occurrence of a variable for better readability):

(26)   John knows the capital that Fred knows.
        A $\exists x_n (x = \iota y_n.(\text{CAP}(y) \land \exists z_m K_f\, z = y) \land \exists v_m K_j\, v = x)$
        B $\exists x_n K_j (x = \iota y_n.(\text{CAP}(y) \land \exists z_m K_f (z = y)))$

The intended readings are captured by assuming the following resolution for the relevant CC-indices:

(27)   a. $n \to \{\text{the capital of Germany, the capital of Italy}, \dots \}$
        b. $m \to \{\text{Berlin, Rome}, \dots \}$

On this resolution, Reading A says that there is a unique capital which Fred can identify by name (the first conjunct in (26)), and that John can identify that capital by name as well (the second conjunct in (26)). On Reading B, John can identify "the capital that Fred knows" with one of the individual concepts in the conceptual cover associated with n. That is, there is some country x such that "the capital that Fred knows" and "the capital of x" denote the same city in all worlds in John's epistemic state.

5 Comparison with classical quantified epistemic logic and the logic of attitude reports

In this section, we compare quantified epistemic logic under conceptual covers (CC) with classical quantified epistemic logic (QEL). Building on results from Aloni (2005b) we show that if there are no shifts of conceptual covers, CC is just classical QEL: the two semantics turn out to define exactly the same notion of validity. Once we allow shifts of covers though, a number of arguably problematic principles which were valid in QEL cease to be valid in CC. All these principles involve variables occurring free in the scope of some epistemic operator. Section 5.3 pro-

vides motivation for the failure of these principles using canonical examples of *de re* attitude reports.

5.1 QEL-validity

We start by defining a classical semantics for the language under consideration. A QEL-*model* for $\mathscr{L}_{CC}$ is a quadruple $\langle W, R, D, I \rangle$ where W, R, D and I are as above. Well-formed expressions are interpreted in models with respect to classical QEL-assignment functions $g \in D^{\mathscr{V}}$ and world $w \in W$. With respect to the semantics defined in section 3.2 we only have to adjust the clauses for variables in Definition 3 and for the existential quantifier in Definition 4.

Definition 6 (QEL-Interpretation of variables). $[\![x_n]\!]_{M,w,g} = g(x_n)$

Definition 7 (QEL-Interpretation of quantification).

$$M, w \models_g \exists x_n \phi \text{ iff } \exists d \in D : M, w \models_{g[x_n/d]} \phi$$

A formula is valid in a QEL-model M iff it is true with respect to all QEL-assignments and all worlds in M. A formula is QEL-valid iff it is valid in all QEL-models.

Definition 8 (QEL-Validity). Let $M = \langle W, R, D, I \rangle$ be a QEL-model for $\mathscr{L}_{CC}$ and ϕ a formula of $\mathscr{L}_{CC}$.

$$
\begin{aligned}
M \models \phi \quad &\text{iff} \quad \forall w \in W, \forall g \in D^{\mathscr{V}} : M, w \models_g \phi \\
\models_{QEL} \phi \quad &\text{iff} \quad \text{for all QEL-models } M : M \models \phi
\end{aligned}
$$

5.2 CC-validity *vs* QEL-validity

Let a CC-model for $\mathscr{L}_{CC}$ containing a single conceptual cover be a classical CC-model, i.e. a CC-model $M = \langle W, R, D, I, C \rangle$ is *classical* iff $|C| = 1$. A formula of $\mathscr{L}_{CC}$ is classically CC-valid iff it is valid in all classical CC-models.

Definition 9 (Classical CC-validity). Let ϕ be a wff in $\mathscr{L}_{CC}$.

$$\models_{CCC} \phi \text{ iff for all CC-models } M : M \text{ is classical} \Rightarrow M \models_{CC} \phi$$

The first result of this section is that if we just consider classical CC-models, the logic of conceptual covers does not add anything to ordinary quantified epistemic logic. Classical CC-validity is just ordinary QEL-validity. This result is expressed by the following proposition. Full proof of this result can be found in Aloni (2005b).

Proposition 1. *Let ϕ be a formula in $\mathscr{L}_{CC}$.*

$$\models_{CCC} \phi \quad iff \quad \models_{QEL} \phi$$

One direction of the proof of this proposition follows from the fact that given a classical CC-model M, we can construct a corresponding QEL-model M' that satisfies the same $\mathscr{L}_{CC}$ formulas as M. Let M be $\langle W, R, D, I, \{CC\} \rangle$. We define the corresponding QEL-model $M' = \langle W', R', D', I' \rangle$ as follows. $W' = W$, $R' = R$, $D' = CC$. For I' we proceed as follows:

(i) $\forall \langle c_1, ..., c_n \rangle \in CC^n$, $w \in W$, $P \in \mathscr{P}$:

$$\langle c_1, ..., c_n \rangle \in I'(P)(w) \quad \text{iff} \quad \langle c_1(w), ..., c_n(w) \rangle \in I(P)(w);$$

(ii) $\forall c \in CC$, $w \in W$, $a \in \mathscr{C}$:

$$I'(a)(w) = c \quad \text{iff} \quad I(a)(w) = c(w).$$

In our construction, we take the elements of the conceptual cover in the old model to be the individuals in the new model, and we stipulate that they do, in all w, what their instantiations in w do in the old model. Clause (i) says that a sequence of individuals is in the denotation of a relation P in w in the new model iff the sequence of their instantiations in w is in P in w in the old model. In order for clause (ii) to be well-defined, it is essential that CC is a conceptual cover, rather than an arbitrary set of concepts. In M', an individual constant a will denote in w the unique c in CC such that $I(a)(w) = c(w)$. That there is such a unique c is guaranteed by the uniqueness condition on conceptual covers. Aloni (2005b) proves the following theorem which shows that this construction works.

Theorem 2. *Let g be a CC-assignment and h a QEL-assignment such that $g = h \cup \{\langle n, CC \rangle \mid n \in N\}$. Let w be any world in W and ϕ any formula in $\mathscr{L}_{CC}$. Then*

$$M, w, g \models_{CC} \phi \text{ iff } M', w, h \models_{QEL} \phi.$$

Now it is clear that if a classical CC-model M and an ordinary QEL-model M' correspond in the way described, then the theorem entails that any formula in $\mathscr{L}_{CC}$ is CC-valid in M iff it is QEL-valid in M'. Thus, given a classical CC-model, we can define an equivalent QEL-model, but also given an QEL-model, we can define an equivalent classical CC-model $\langle W, R, D, I, \{CC\} \rangle$ by taking CC to be the 'rigid' cover $\{c \in D^W \mid \exists d. \forall w. c(w) = d\}$. This suffices to prove Proposition 1.

A corollary of Proposition 1 is that CC-validity is weaker than QEL-validity. $\models_{CC} \phi$ obviously implies $\models_{CCC} \phi$ which by Proposition 1 implies $\models_{QEL} \phi$.

Corollary 1. *If $\models_{CC} \phi$, then $\models_{QEL} \phi$.*

A further consequence of Proposition 1 is that we can define interesting fragments of $\mathscr{L}_{CC}$ which behave classically, that is, formulas in these fragments are CC-valid iff they are valid in QEL. This is done in the following propositions.

Proposition 2. *Let $\mathscr{L}_{CC}^n$ be a restriction of $\mathscr{L}_{CC}$ containing only variables indexed by n, and $\phi \in \mathscr{L}_{CC}^n$. Then $\models_{CC} \phi$ iff $\models_{QEL} \phi$.*

proof: Suppose $\not\models_{CC} \phi$ for $\phi \in \mathscr{L}_{CC}^n$. This means for some *CC*-model $M = \langle W, R, D, C, I \rangle$ and some w,g: $M, w \not\models_g \phi$. Let $M' = \langle W, R, D, \{g(n)\}, I \rangle$. Since ϕ can only contain variables indexed by n, $M', w \not\models_g \phi$. M' is obviously a classical model. This means $\not\models_{CCC} \phi$ which by Proposition 1 implies $\not\models_{MPL} \phi$. Corollary 1 delivers the second half of Proposition 2.   □

Proposition 3. *Let $\mathscr{L}_{PL}$ be the non-modal fragment of $\mathscr{L}_{CC}$, and $\phi \in \mathscr{L}_{PL}$. Then $\models_{CC} \phi$ iff $\models_{QEL} \phi$.*

proof: Suppose $\not\models_{CC} \phi$. This means for some *CC*-model $M = \langle W, R, D, C, I \rangle$ and some w,g: $M, w \not\models_g \phi$. Let $M' = \langle W', R', D, C', I' \rangle$, be a sub-model of M such that $W' = \{w\}$. Since ϕ is non-modal $M', w \not\models_g \phi$. Since $|W'| = 1$, $|C'| = 1$, i.e. M' is a classical model. This means $\not\models_{CCC} \phi$ which by Proposition 1 implies $\not\models_{MPL} \phi$. Again Corollary 1 delivers the other direction of the proof.   □

   The following proposition, which is a novel result with respect to (Aloni, 2005b), shows that conceptual covers only play a role in cases of 'quantifying in', i.e., when we have a variable occurring free in the scope of a modal operator.

   Let ϕ be a formula in $\mathscr{L}_{CC}$. Then ϕ is *closed* iff no variable occurs free within the scope of a modal operator in ϕ. The following proposition states that in such cases the interpretation of ϕ is independent of the conceptual cover parameter irrespective of the number of modal operators or *CC*-indices the formula contains.

Proposition 4. *Let ϕ be a closed formula in $\mathscr{L}_{CC}$. Then $\models_{CC} \phi$ iff $\models_{QEL} \phi$.*

One direction of the proof of this proposition follows from Corollary 1. The other direction follows from the fact that given a CC-model $M = \langle W, R, D, I, C \rangle$, a world $w \in W$ and a CC-assignment g, we can define a corresponding QEL-model $M' = \langle W, R, D, I \rangle$, and QEL-assignment $h_{g,w}$, defined as the element of $D^{\mathscr{V}_N}$ such that for all $v \in \mathscr{V}_N, h(v) = g(v)(w)$, for which we can prove the following theorem for all closed ϕ:

Theorem 3.

$$M, w, g \models_{CC} \phi \text{ iff } M', w, h_{g,w} \models_{QEL} \phi.$$

So whenever we can invalidate a closed ϕ in *CC* we will be able to invalidate ϕ in QEL as well (see appendix for proof).

5.3 On the logic of attitude reports

As a consequence of Proposition 3, our CC semantics validates the principles of existential generalisation and substitutivity of identicals for non-modal formulas, since they are validated in QEL:

SI1 $\models_{CC} t = t' \rightarrow (\phi[t] \rightarrow \phi[t'])$ (if ϕ is non-modal)
EG1 $\models_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$ (if ϕ is non-modal)

Note that the validity of **EG1** crucially relies on the existence condition on conceptual covers, which guarantees that whatever denotation, $d = [\![t]\!]_{M,g,w}$, a term t is assigned in w, there is a concept c in the operative cover such that $c(w) = d = [t]_{M,g,w}$.

Substitutivity of identicals and existential generalisation cease to hold as soon as we introduce modal operators. By Corollary 1, **SI** and **EG** are invalidated in CC, being invalid in QEL:

SI $\not\models_{CC} t = t' \rightarrow (\phi[t] \rightarrow \phi[t'])$
EG $\not\models_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$

The failure of **SI** allows us to handle, as in (Hintikka, 1962), the canonical failures of substitutivity of identicals in the scope of propositional attitudes:

(28)   a.   Hesperus is Phosphorus. The Babylonians knew that Hesperus is Hesperus. $\not\Rightarrow$ The Babylonians knew that Hesperus is Phosphorus.
         b. $\not\models_{CC} t = t' \rightarrow (\Box t = t \rightarrow \Box t = t')$

The failure of **EG** allows us to preserve important aspects of Hintikka's original account of the contrast between *de re* and *de dicto* attitude ascription, under the assumption that the pragmatic resolution of CC-indices is governed by general Gricean conversational principles (see Aloni, 2005a, for a formalisation of such a pragmatic theory):

(29)   a.   Ralph believes that the shortest spy is a spy $\not\Rightarrow$ Ralph believes someone to be a spy.   (Quine, 1956; Kaplan, 1969)
         b. $\not\models_{CC} \Box Pt \rightarrow \exists x_n \Box Px_n$

In Hintikka's semantics, existential generalisation can be applied to t only if t is a rigid designator. The present semantics is more liberal: $\exists x_n \Box \text{SPY}(x_n)$ follows from $\Box \text{SPY}(\text{THE-SHORTEST-SPY})$, if we map n into a cover which includes the individual concept $\lambda w [\![\text{THE-SHORTEST-SPY}]\!]_w$. Such a resolution however is blocked in ordinary conversations because it would involve a violation of Grice's maxim of Quantity.

In contrast to what happens in Hintikka's QEL, not only **SI** and **EG** are invalidated in the present semantics, but also **SIv** and **EGv**.

SIv $\not\models_{CC} x_n = y_m \rightarrow (\phi[x_n] \rightarrow \phi[y_m])$
EGv $\not\models_{CC} \phi[y_m] \rightarrow \exists x_n \phi[x_n]$

From the failure of **SIv**, it follows that also **LIv** is not valid in CC.

LIv $\not\models_{CC} x_n = y_m \rightarrow \Box \, x_n = y_m$

From the failure of **EGv**, it follows that also the principle of renaming **PR** is not generally valid in CC:

PR $\not\models_{CC} \exists x_n \Box P(x_n) \leftrightarrow \exists y_m \Box P(y_m)$

These related invalidities allow us to model a number of cases of mistaken identity that were problematic in Hintikka's original semantics. Recall Quine's Ralph who believes of one man that he is two distinct individuals, because he has seen him in two different circumstances, once with a brown hat, once at the beach. In Quine's story Ralph suspects that the man with the brown hat is a spy, while he thinks that the man seen at the beach is rather a pillar of the community. "Can we say of this man (Bernard J. Ortcutt to give him a name) that Ralph believes him to be a spy?" (Quine, 1956, p. 179). In the CC-semantics described above, we can give a reasonable answer to Quine's question, namely, "It depends". The question receives a negative or a positive answer relative to the way in which Ortcutt is specified. The following formula is true under a CC-assignment that maps x_n to a concept c_1 corresponding to "the man with the brown hat" and false under a CC-assignment that maps x_n to a concept c_2 corresponding to "the man on the beach".

(30) $\Box Sx_n$

Notice that c_1 and c_2 cannot be part of one and the same conceptual cover because they assign one and the same individual to the actual world, so uniqueness would be violated. As a consequence of this, the following two sentences can both be true even in a serial model, but only if n and m are assigned different conceptual covers.

(31)   a.   Ralph believes Ortcutt to be a spy.
         b. $\exists x_n(x_n = o \wedge \Box Sx_n)$

(32)   a.   Ralph believes Ortcutt not to be a spy.
         b. $\exists x_m(x_m = o \wedge \Box \neg Sx_m)$

This is intuitively reasonable: one can accept these two sentences without drawing the conclusion that Ralph's beliefs are inconsistent, only if one takes into consideration the two different perspectives under which Ortcutt can be considered. Furthermore, the fact that a shift of cover is required in this case explains the never ending puzzling effect of Quine's story. After reading Quine's description of the facts, both covers (the one identifying Ortcutt as the man with the brown hat, the other identifying Ortcutt as the man on the beach) are equally salient, and this causes bewilderment in the reader who has to choose one of the two in order to interpret each *de re* sentence.

From (31) and (32) we *cannot* infer the following (for $i \in \{n, m\}$):

(33) $\exists x_i(x_i = o \wedge \Box(Sx_i \wedge \neg Sx_i))$

which would charge Ralph with contradictory beliefs. Yet, we can infer (34) which does not carry such a charge:

(34) $\exists x_n(x_n = o \wedge \exists y_m(o = y_m \wedge \Box(Sx_n \wedge \neg Sy_m)))$

Note finally that substitutivity of identicals and existential generalisation are allowed when applied to variables with a uniform index. It is easy to see that the present semantics validates the following schemes:

SIn $\models_{CC} x_n = y_n \rightarrow (\phi[x_n] \rightarrow \phi[y_n])$
EGn $\models_{CC} \phi[y_n] \rightarrow \exists x_n \phi[x_n]$

The validity of **SIn** crucially relies on the uniqueness condition on conceptual covers. From **SIn**, but also as a consequence of Proposition 2, we can derive **LIn**, which guarantees that the elements in our domains of quantification behave more like individuals than representations:

LIn $\models_{CC} x_n = y_n \rightarrow \Box x_n = y_n$

In this section we have seen that quantified epistemic logic under conceptual covers is essentially richer than classical QEL because in the former we can shift from one cover to another, and these shifts can affect evaluation of formulas containing variables occurring free in the scope of some epistemic operator. If we stick to one cover, or we restrict attention to closed formulas, then not only do CC and QEL define the same notion of validity (Proposition 1 and Proposition 4), but also, and maybe more significantly, the same notion of truth (Theorem 2 and Theorem 3). We have already seen the intuitive consequences of this result. On the one hand, in ordinary situations in which the method of identification is kept constant, CC behaves exactly like QEL and inherits its desirable properties (for example in relation to the shortest spy example). On the other hand, the system is flexible enough to account for extraordinary situations as well, such as Quine's double vision situations, where multiple covers are operative.

6 Conclusion

Hintikka taught us how to analyse knowing-who constructions in quantified epistemic logic: a knows who t is ($\exists x \Box x = t$) iff t denotes one and the same individual in all of a's epistemic alternatives. This article has proposed the following generalisation of Hintikka's analysis: a knows who$_n$ t is ($\exists x_n \Box x_n = t$) iff t can be identified by one and the same concept in the contextually selected cover n in all of a's epistemic alternatives.

This generalisation allows a ready account of (i) the context-sensitivity of knowing-who constructions (Boër and Lycan, 1985), acknowledged by Hintikka himself; (ii) quantified and embedded cases of concealed questions (Heim, 1979) and (iii) canonical problematic examples of failure of substitutivity of identicals in attitude reports (Frege, 1892; Quine, 1956; Kaplan, 1969).

Appendix

Let $M = \langle W, R, D, I, C \rangle$ be a CC-model and $M' = \langle W, R, D, I \rangle$ be the corresponding classical QEL-model. And given a CC-assignment g and a world $w \in W$, let $h_{g,w}$ be an element of $D^{\mathcal{V}_N}$ such that for all $v \in \mathcal{V}_N : h(v) = g(v)(w)$. We prove the following theorem for any closed ϕ in $\mathscr{L}_{CC}$:

Theorem 3.

$$M, w, g \models_{CC} \phi \text{ iff } M', w, h_{g,w} \models_{QEL} \phi.$$

Proof: The proof is by induction on the construction of ϕ. We start by showing that the following holds for all terms t:

(A) $[\![t]\!]_{M,w,g} = [\![t]\!]_{M',w,h_{g,w}}$.

Suppose t is a variable. Then $[\![t]\!]_{M,w,g} = g(t)(w)$. By definition of $h_{g,w}$, $g(t)(w) = h_{g,w}(t)$, which means that $[\![t]\!]_{M,w,g} = [\![t]\!]_{M',w,h_{g,w}}$. Suppose now t is a constant. Then $[\![t]\!]_{M,w,g} = I(t)(w) = [\![t]\!]_{M',w,h_{g,w}}$. We can now prove the theorem for atomic formulae.

Suppose ϕ is $Pt_1, ..., t_n$. Now $M, w, g \models_{CC} Pt_1, ..., t_n$ holds iff (a) holds:

(a) $\langle [\![t_1]\!]_{M,w,g}, ..., [\![t_n]\!]_{M,w,g} \rangle \in I(P)(w)$.

By (A), (a) holds iff (b) holds:

(b) $\langle [\![t_1]\!]_{M',w,h_{g,w}}, ..., [\![t_n]\!]_{M',w,h_{g,w}} \rangle \in I(P)(w)$,

which means that $M', w, h_{g,w} \models_{QEL} Pt_1, ..., t_n$.

Suppose now ϕ is $t_1 = t_2$. $M, w, g \models_{CC} t_1 = t_2$ holds iff (c) holds:

(c) $[\![t_1]\!]_{M,w,g} = [\![t_2]\!]_{M,w,g}$.

By (A) above, (c) holds iff (d) holds:

(d) $[\![t_1]\!]_{M',w,h_{g,w}} = [\![t_2]\!]_{M',w,h_{g,w}}$,

which means that $M', w, h_{g,w} \models_{QEL} t_1 = t_2$.

Suppose now ϕ is $\Box \psi$. $M, w, g \models_{CC} \Box \psi$ holds iff (e) holds:

(e) $\forall w' \in W : wRw' : M, w', g \models_{CC} \psi$.

By induction hypothesis, (e) holds iff (f) holds:

(f) $\forall w' \in W : wRw' : M', w', h_{g,w'} \models_{QEL} \psi$.

And (f) holds iff (g) holds:

(g) $M', w, h_{g,w'} \models_{QEL} \Box \psi$.

Since ψ does not contain any free variable, (g) is equivalent to $M',w,h_{g,w} \models_{QEL} \Box\psi$.

Suppose now ϕ is $\exists x_n\psi$. $M,w,g \models_{CC} \exists x_n\psi$ holds iff (h) holds:

(h) $\exists c \in g(n) : M,w,g[c/x_n] \models_{CC} \psi$.

By induction hypothesis, (h) holds iff (i) holds:

(i) $\exists c \in g(n) : M',w,h_{g[x_n/c],w} \models_{QEL} \psi$.

By definition $h_{g[x_n/c],w} = h_{g,w}[x_n/c(w)]$, and $c(w) \in D$. But then (i) holds iff (j) holds:

(j) $\exists d \in D : M',w,h_{g,w}[x_n/d] \models_{QEL} \psi$,

which means $M',w,h_{g,w} \models_{QEL} \exists x_n\psi$.

The induction for $\neg$ and $\wedge$ is immediate. $\Box$

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