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Free choice, modals, and imperatives

Maria Aloni

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Abstract The article proposes an analysis of imperatives and possibility and necessity statements that (i) explains their differences with respect to the licensing of free choice *any* and (ii) accounts for the related phenomena of free choice *disjunction* in imperatives, permissions, and other possibility statements. *Any* and *or* are analyzed as operators introducing sets of alternative propositions. Free choice licensing operators are treated as quantifiers over these sets. In this way their interpretation can be sensitive to the alternatives *any* and *or* introduce in their scope.

Keywords Free choice · Indefinites · Disjunction · Alternatives · Modals · Imperatives

1 Introduction

This article discusses the distribution and interpretation of *any* and *or* in modal statements and imperatives. It has often been observed that *any* and *or* have a common character: in any context in which the former is licensed and

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M. Aloni (✉)
Faculty of Philosophy, University of Amsterdam,
Nieuwe Doelenstraat 15,
1012 CP Amsterdam, The Netherlands
e-mail: M.D.Aloni@uva.nl

receives a free choice interpretation, the latter can give rise to non-standard inference patterns (Horn 1972; Kamp 1973). As an illustration consider the following examples.

(1) a. Vincent may be anywhere.
b. #Vincent is anywhere.
c. #Vincent must be anywhere.

(2) a. Vincent may be in Paris or in London. $\Rightarrow$
Vincent may be in Paris *and* Vincent may be in London
b. Vincent is in Paris or in London. $\not\Rightarrow$
Vincent is in Paris and Vincent is in London.
c. Vincent must be in Paris or in London. $\not\Rightarrow$
Vincent must be in Paris and Vincent must be in London.

Any is licensed in possibility statement (1a), but it is out in episodic and necessity statements (1b, c). In a parallel fashion, *or* can give rise to an unexpected inference patterns in (2a), but not in (2b, c). In (1a), *any* yields a free choice interpretation. The sentence can be paraphrased as 'Whatever location you choose, Vincent may be there'. Example (2a), on its most prominent reading, conveys the same free choice effect ('Whichever location you choose, be it Paris or London, Vincent may be there').[1]

Imperative sentences appear to pattern with possibility statements, as illustrated in examples (3) and (4). *Any* is licensed in (3) and receives a free choice interpretation ('You may choose which key').

(3) To continue push any key!

As observed by Giannakidou (2001), example (3) is not an instruction to push *all* keys. Free choice *any* then, which in possibility statements like (1a) yielded a universal-like interpretation, behaves here like a 'genuine' existential quantifier (*contra* e.g. Dayal 1998).

Example (4) shows that the law of propositional logic that states the deducibility of *either A or B* from *A* fails to hold for imperatives (Ross's (1941) paradox).

(4) a. Post this letter! $\not\Rightarrow$ b. Post this letter or burn it!

The most natural interpretation of disjunctive imperatives like (4b) is as one presenting a choice between different actions. Imperative (4a) then cannot imply (4b); otherwise, when told the former, I would be justified in burning the letter rather than posting it.

The phenomena in (1)–(4) constitute a problem for prominent theories of free choice and disjunction. Kadmon and Landman's (1993) elegant analysis

[1] Example (2a) also has another reading, sometimes called 'wide scope *or*' reading, which can be brought out by appending to the sentence the continuation: "..., but I don't know which." On the latter reading the sentence does not have any free choice implication.

of *any* as an indefinite with a generic meaning, if combined with standard treatments of modals and imperatives, fails to explain (1) and (3). Moreover, as is well known, the standard Boolean account of *or* as set union leaves the facts in (2) and (4) unaccounted for. Recent approaches have attempted to solve some of these problems by adopting different analyses for free choice *any* (Dayal 1998; Giannakidou 2001) and *or* (Zimmermann 2000, Simons 2005, Geurts 2005). Although these attempts are very interesting and manage to capture important generalizations, the analysis I would like to defend in this article follows a different strategy. I propose to maintain Kadmon and Landman's analysis of *any* as an existential quantifier ($\exists$), and the treatment of *or* as logical disjunction ($\vee$). I assume, however, that beyond their standard truth-conditional contributions, $\exists$ and $\vee$ also have the potential to introduce sets of propositional alternatives. Modals and imperatives are then held to be operators over these sets of alternatives. The resulting analysis gives us a unified account of the phenomena in (1)–(4).

Sets of propositional alternatives are widely held to play a role in the semantics of questions and focus (Hamblin 1973; Karttunen 1977; Groenendijk and Stokhof 1984; Rooth 1992). The idea at the heart of my proposal is that propositional alternatives may enter the recursive characterization of the semantics of a much wider range of natural language expressions, including (a) indefinites and disjunctions, held to generate these sets of alternatives; and (b) operators like modals or imperatives, held to quantify over the sets of alternatives generated in their scope.[2]

The article is structured as follows. The next section reviews prominent theories of free choice items, modals, and imperatives and discusses their problems. Section 3 presents a semantics, inspired by the analysis of questions, which accounts for the alternative propositions introduced by occurrences of $\exists$ and $\vee$. Section 4 proposes an analysis of modals as quantifiers over these alternative propositions and discusses a number of applications. Section 5 extends this analysis to imperative sentences. Finally, Section 6 draws conclusions and lists a number of further lines of research.

2 Some background

2.1 *Any*: Kadmon and Landman (1993)

English employs *any* in two different ways. *Any* can function as a negative polarity item, and it can obtain a free choice interpretation. In an influential article, Nirit Kadmon and Fred Landman (henceforth K&L) have proposed a unified analysis of polarity sensitive and free choice (henceforth FC) *any*, where a

[2] The 'Hamblin semantics' for indefinite pronouns put forward in Kratzer and Shimoyama (2002) also uses sets of propositional alternatives. Since the present article was originally written, very interesting accounts of free choice *any* (Menéndez-Benito 2005) and disjunction (Alonso-Ovalle 2006) have been proposed in that framework. A proper comparison with my analysis, however, must be left to another occasion.

phrase '*any* CN' is uniformly treated as an indefinite expression with two additional semantic/pragmatic characteristics: widening and strengthening.

To motivate the first characteristic, K&L consider the following example:

(5) A: Do you have dry socks?
B: I don't have ANY socks.

K&L observe that the use of *any* in B's reply conveys that wet socks are no exception to her claim that she doesn't have socks. The widening condition is meant to capture this 'reduced tolerance of exceptions'.

WIDENING: *Any* widens the interpretation of the common noun along a contextual parameter.

On K&L's account, *any* is an existential quantifier that widens the domain which otherwise would be associated with it by the context of utterance. This widening must occur for a reason though, which explains why *any* is so picky in its distribution. The reason that K&L propose for the domain widening of *any* is the strengthening of the statement made. In conversation, if given a choice, we normally go for the most informative candidate. It is only in structures in which domain widening leads to a stronger statement that, according to K&L, *any* is allowed. This leads us to the second characteristic of *any*.

STRENGTHENING: *Any* is licensed only if the domain widening that it induces creates a stronger statement.

The strength of a sentence is defined in terms of entailment. Strengthening means that *any* is licensed only if the statement on the wide interpretation entails the statement on the narrow interpretation.

Let us see how K&L's analysis successfully captures the basic generalizations about *any*.

The first example concerns an episodic sentence. Let A and B be contextually selected quantificational domains such that $A \supseteq B$.

(6) #John talked to any student.
a. *wide*: $\exists_A x(Sx \wedge Tjx)$ $\not\Rightarrow$
b. *narrow*: $\exists_B x(Sx \wedge Tjx)$

K&L correctly predict that *any* is not licensed in (6), because enlarging the domain of the existential in this construction leads to a loss of information.[3]

[3] It has long been observed that sentences like (6) can be rescued by the addition of a postnominal modifier, as in *John talked to any student that came up to him*. Licensing by a modifier is often called *subtrigging* since Dayal's (1998) revival of this term originally from LeGrand. If subtrigged sentences are episodic sentences, as is generally claimed in the literature (Dayal 1998), K&L's analysis and the one defended in the present article won't be able to account for this phenomenon. See (Menéndez-Benito 2005; Aloni 2006b) for possible ways out.

In negative contexts, we obtain the opposite result. Since negation reverses entailment, domain widening leads to stronger negative sentences and, therefore, (7) is correctly predicted to be grammatical.

(7) John did not talk to any student.
a. *wide*: $\neg\exists_A x(Sx \wedge Tjx) \quad \Rightarrow$
b. *narrow*: $\neg\exists_B x(Sx \wedge Tjx)$

Under negation, *any* is licensed. We talk in these contexts of a negative polarity interpretation.

The last example concerns FC *any* in a generic sentence. Let GEN_x stand for a generic operator, the interpretation of which is assumed to change the quantificational force of $\exists$ in its scope from existential to universal in much the same way as in standard dynamic accounts of (un)selective binding (e.g. Dekker's 1993 analysis of adverbial quantification).

(8) Any dog hunts cats.
a. *wide*: $\text{GEN}_x(\exists_A xDx; HCx) \quad \Rightarrow$
b. *narrow*: $\text{GEN}_x(\exists_B xDx; HCx)$

K&L correctly predict the felicity of (8). Domain widening leads to a stronger statement here because of the effect of the generic operator that binds the indefinite and thereby gives it universal force.

To conclude, in the K&L analysis, polarity sensitive and FC *any* are uniformly treated as existential quantifiers. The universal effect of FC *any* is the result of binding by an operator with universal force, for example a generic operator. On this analysis, FC *any* is basically an indefinite interpreted generically.

In the next sections, we will see whether K&L's theory and the standard Boolean analysis of disjunction extend to explain *any* and *or* in modal contexts and imperatives. Let us start by reviewing what *may* and *must* are normally taken to mean.

2.2 Modals: the standard account

On a standard account of modal expressions, *may* (or *can*) and *must* are analyzed in terms of compatibility and entailment with respect to a set of possible worlds which varies relative to the sort of modality under discussion (epistemic, deontic, ...) and other pragmatic factors (Kratzer 1977).

(i) MAY ϕ is true in w iff ϕ is *compatible* with the relevant set of worlds;

(ii) MUST ϕ is true in w iff ϕ is *entailed* by the relevant set of worlds.

Two problems arise if we assume this analysis. First, in combination with K&L's theory of *any*, it fails to predict the felicity of examples like (9a, b).

(9)a. You may pick any flower.
b. Any pilot could be flying this plane.

As Dayal (1998) observed, the ordinary indefinite counterparts of these sentences have existential interpretations.

(10)a. You may pick a flower.
b. A pilot could be flying this plane.

This excludes the possibility to derive the free choice effects of (9a, b) from the presence of a generic operator. But then (9a, b) are predicted to be bad. Domain widening never strengthens a plain existential possibility statement, regardless of whether $\exists$ takes narrow or wide scope over the modal operator.

(11) a. $\text{MAY}(\exists x \phi(x))$ widening $\not\Rightarrow$ strengthening
b. $\exists x(\text{MAY}(\phi(x)))$ widening $\not\Rightarrow$ strengthening

Second, this analysis of modals, when combined with a Boolean analysis of *or*, leaves the phenomenon of free choice disjunction in possibility statements unaccounted for. As is easy to see, (12c), which analyzes sentence (12a), does not entail (12d), which analyses (12b).

(12) a. John or Mary may attend the meeting. $\Rightarrow$
b. John may attend the meeting and Mary may attend the meeting.
c. $\text{MAY}(\phi(j) \vee \phi(m)) \quad \not\Rightarrow$
d. $\text{MAY}(\phi(j)) \wedge \text{MAY}\ (\phi(m))$

Before proposing a solution to these problems, I will discuss the case of *any* and *or* in imperative sentences. Unfortunately, there is no standard theory of imperatives we can assume here. The next section presents a minimal account of the meaning of imperative sentences and shows its difficulties to explain the phenomena of free choice illustrated in the introductory examples (3) and (4).

2.3 Imperatives: a minimal account

Declaratives have truth conditions. Interrogatives have answerhood conditions. Imperatives, on the other hand, have *compliance conditions*. Someone cannot be said to understand the meaning of an imperative $!\phi$ unless she recognizes what has to be true for the command (or request, advice, etc.) issued by utterance of $!\phi$ to be complied with. A natural way to account for this intuition is to identify the compliance conditions of imperative $!\phi$ with the proposition expressed by ϕ.[4] For example, disregarding tense, in the following two examples the imperatives in (a) can be taken to express the compliance conditions in (b).[5]

[4] But see Portner (2004) or Mastop (2005) who, among others, have argued that imperatives are better analyzed in terms of properties or actions rather than propositions.

[5] To explain where the subjects of the propositions in (b) originate, we can assume the presence of a covert addressee-referring pronoun in the imperatives in (a). Evidence for this comes from languages like Italian, which show agreement morphology within imperatives. For example, imperative *Shut the door!* translates in Italian as *Chiudi la porta!* for 2nd person singular subject, but as *Chiudete la porta!* for 2nd person plural subject.

(13) a. Post this letter!
b. That the hearer posts the letter.

(14) a. Post this letter or burn it!
b. That the hearer posts the letter or the hearer burns the letter.

Entailment between imperatives can then be defined in terms of inclusion of their compliance conditions. The intuition is that an imperative entails another iff each way of complying with the former is a way of complying with the latter. By this analysis we would capture, for example, the inference patterns illustrated in (15).

(15) a. Kill everybody! $\Rightarrow$ Kill Bill!
b. Kill somebody/anybody! $\not\Rightarrow$ Kill Bill!

Given these assumptions, however, we fail to explain Ross's paradox. The proposition in (13b) is included in (14b). Therefore, (13a) would entail (14a). Second, imperatives like (16a) would express plain existential propositions since their ordinary indefinite counterparts do not have generic interpretations. But then K&L's widening and strengthening conditions fail to account for their potential to license *any* in their scope.

(16) a. Push any key! 'that the hearer pushes a key'
b. $!\exists x\phi(x)$ widening $\not\Rightarrow$ strengthening

To conclude, K&L's account of *any* and a Boolean account of *or* fail when combined with minimal accounts of modals and imperatives. The following sections propose a unified solution to these difficulties.

3 The logic of alternatives

The starting point of my proposal is the observation of the common character of *any* and *or* reflected by their formal counterparts $\exists$ and $\vee$. As is clear from the following specification of the truth conditions of these constructions, existentially quantified sentences and disjunctions express that at least one element of a larger set of propositions is true, but do not specify which. (By $[\![\phi]\!]_{M,w,g}$ and $[\![\phi]\!]_{M,g}$ I denote the extension (truth value) and intension (proposition, i.e. set of possible worlds) of ϕ in model M with respect to (world w and) assignment g respectively.)

$$\begin{array}{lll} [\![\exists x A(x)]\!]_{M,w,g} = 1 & \Leftrightarrow & \exists p \in \{[\![A(x)]\!]_{M,g[x/d]} \mid d \in D\} : w \in p; \\ [\![A \vee B]\!]_{M,w,g} = 1 & \Leftrightarrow & \exists p \in \{[\![A]\!]_{M,g}, [\![B]\!]_{M,g}\} : w \in p. \end{array}$$

Both $\exists xA$ and $A \vee B$ can be thought of as introducing a set of alternative propositions and, indirectly, raising the question about which of these alternatives is true. In this section, I give a formal account of the sets of propositional alternatives introduced by these constructions. I will then show how this logic of alternatives is needed for a proper analysis of interrogative sentences.

3.1 Formal definitions

The logic of alternatives introduced in this section is a version of modal predicate logic with propositional quantifiers (building on Fine 1970) where satisfaction is defined with respect to propositional witness sequences (building on Dekker 2002).

The language is that of standard modal predicate logic with the addition of propositional quantifiers and propositional identity, so that, for example, we can write $\exists p(p \wedge p = A)$ for A.

A model M is a quintuple (W, R, P, D, I), where W is a non-empty set of worlds, R is an accessibility relation, P is a non-empty set of subsets of W (i.e. of propositions) satisfying a number of properties (Fine 1970), D is a non-empty domain of individuals, and I is an interpretation function for the non-logical part of the language. An assignment function g maps individual variables x to elements of D, and propositional variables p to elements of P.

The semantics is spelled out in terms of a satisfaction relation $\models_g$, which may hold between a model M, a world w, and a sequence s of witnesses from P, on the one hand, and a formula ϕ, on the other. In the definition we also take into account what is referred to as $n(\phi)$, the number of 'surface' existential propositional quantifiers in ϕ (Dekker 2002).

Definition 1

$n(Rx_1 \ldots x_n)$	$= 0$	$n(\phi \wedge \psi)$	$=$	$n(\phi) + n(\psi)$
$n(\neg\phi)$	$= 0$	$n(\exists x\phi)$	$=$	$n(\phi)$
$n(\diamond\phi)$	$= 0$	$n(p)$	$=$	0
$n(\phi = \psi)$	$= 0$	$n(\exists p\phi)$	$=$	$1 + n(\phi)$

This notion counts the number of active existential propositional quantifiers in the sentence. All connectives, except for conjunction and the existential individual quantifier, deactivate any occurrence of propositional quantifiers in their scope. Satisfaction is defined as follows:[6]

Definition 2 (Satisfaction)

$M, w, s \models_g Rx_1, ..., x_n$	iff	$\langle g(x_1), ..., g(x_n)\rangle \in I(R)(w)$
$M, w, s \models_g \neg\phi$	iff	$M, w, cs \models_g \phi$, for no $c \in P^{n(\phi)}$
$M, w, cs \models_g \phi \wedge \psi$	iff	$M, w, s \models_g \phi$ & $M, w, cs \models \psi$, for $c \in P^{n(\psi)}$
$M, w, s \models_g \exists x\phi$	iff	$M, w, s \models_{g[x/d]} \phi$, for $d \in D$
$M, w, s \models_g \diamond\phi$	iff	$\exists v : wRv$ & $M, v, cs \models_g \phi$, for $c \in P^{n(\phi)}$
$M, w, s \models_g p$	iff	$w \in g(p)$
$M, w, s \models_g \phi = \psi$	iff	$\forall v : M, v, cs \models_g \phi$ iff $M, v, ds \models_g \psi$, for $c \in P^{n(\phi)}, d \in P^{n(\psi)}$
$M, w, qs \models_g \exists p\phi$	iff	$M, w, s \models_{g[p/q]} \phi$, for $q \in P$

[6] Eventually, in order to express domain widening, individual quantifiers will have to be indexed to a contextually selected domain (Westerståhl 1984).

Disjunction $\vee$, implication $\rightarrow$, universal quantification $\forall$, and necessity $\Box$ are defined as standard in terms of $\neg$, $\wedge$, $\exists$, and $\Diamond$. I spell out the clause for disjunction because it plays a crucial role below:

$$M, w, s \models_g \phi \vee \psi \text{ iff } M, w, cs \models_g \phi \text{ or } M, w, ds \models_g \psi, \text{ for } c \in P^{n(\phi)}, d \in P^{n(\psi)}$$

Truth and entailment are defined as follows.

Definition 3 (Truth and entailment)

(i) $M, w \models_g \phi$ iff $\exists s : M, w, s \models_g \phi$;
(ii) $\phi \models \psi$ iff $\forall M, \forall w, \forall g : M, w \models_g \phi \Rightarrow M, w \models_g \psi$.

The most interesting clause in Definition 2 is that of the propositional existential quantifier. Sentence $\exists p\phi$ is satisfied in w relative to s iff the first element in s is a witness of the truth of the sentence in w. For example, given the interpretation of propositional variables, sentence $\exists p\, p$ is satisfied in w wrt qs iff $w \in q$. Given the interpretation of identity, $\exists p(p \wedge p = A)$ is satisfied in w wrt qs iff $w \in q$ and q is the proposition expressed by A. And finally, given the clause for disjunction, and for the existential individual quantifier, $\exists p(p \wedge (p = A \vee p = B))$ is satisfied in w wrt qs iff $w \in q$, and q is the proposition expressed by A or q is the proposition expressed by B; and $\exists p(p \wedge \exists x(p = A(x)))$ is satisfied in w wrt qs iff $w \in q$, and q is the proposition expressed by $A(x)$ for some value of x.

The core idea of the formalization is that sentences are mapped to *structured* propositions, rather than plain propositions:

(17) a. Structured propositions: $\lambda s \lambda w : M, w, s \models_g \phi$
b. Standard propositions: $\lambda w : M, w \models_g \phi$

This extra structure is used to derive the proper set $\textsc{alt}(\phi)_{M,g}$ of propositional alternatives induced by ϕ.

Definition 4 (Alternative sets)

$\textsc{alt}(\phi)_{M,g} = \{\{w \mid M, w, s \models_g \phi\} \mid s \in P^{n(\phi)}\} \setminus \emptyset$

The propositional alternatives introduced by a sentence are defined in terms of the set of possible values for an existentially quantified propositional variable. If $n(\phi) = 0$, i.e. if ϕ does not contain any surface occurrence of an existential propositional quantifier, then $\textsc{alt}(\phi)_{M,g}$ is a singleton set containing the standard proposition expressed by ϕ. In case $n(\phi) \neq 0$, the possibility of a multi-membered alternative set arises. If $n(\psi) = 0$, then $\exists p\psi(p)$ induces the set $\{$that q_1 is such that ψ, that q_2 is such that ψ, ... $\}$. If ψ expresses a property holding of more than one proposition, then $\textsc{alt}(\exists p\psi(p))_{M,g}$ will be a set of genuine alternatives. This holds, for example, for (18a), but not for (18b):

(18) a. $\textsc{alt}(\exists pp)_{M,g} = P$
b. $\textsc{alt}(\exists p(p \wedge p = r))_{M,g} = \{g(r)\}$

The sentence in (18b), $\exists p(p \wedge p = r)$, and r are truth conditionally equivalent. They also induce the same alternative set. The use of a propositional quantifier does not add anything in this case. More interesting possibilities arise, instead, when propositional quantification interacts with the individual quantifier $\exists$ or with $\vee$. Suppose you want to express an existential proposition or a disjunction. The logic now offers you the following options:

(19)a. $\exists x A(x) / \exists p(p \wedge p = \exists x A(x))$ a′. $\boxed{\exists x A(x)}$

b. $\exists p(p \wedge \exists x(p = A(x)))$ b′. $\boxed{A(d_1)}$ $\boxed{A(d_2)}$ $\boxed{\ldots}$

(20)a. $A \vee B / \exists p(p \wedge p = A \vee B)$ a′. $\boxed{A \vee B}$

b. $\exists p(p \wedge (p = A \vee p = B))$ b′. $\boxed{A}$ $\boxed{B}$

Although the (a) and (b) sentences are truth conditionally equivalent, the sets of alternatives they bring about, depicted on their right, are not the same. While the (a) representations introduce singleton sets, the (b) representations induce genuine sets of alternatives.[7]

3.2 Independent motivation

On this account, a sentence, beyond having truth conditions, also introduces a set of propositional alternatives. In this section, I briefly show that this extra structure is needed for a proper account of interrogative sentences.

Let interrogatives $?\phi$ denote the sets of alternatives induced by ϕ. Then the sets induced by (19a) and (19b) above can serve as denotations for polar existential questions ((21)) and constituent questions ((22)), respectively.

[7] See the Appendix for the derivation of the alternative sets generated by these sentences.

(21)a. Does anybody smoke?

b. $?\exists x A(x)$ b′. $\boxed{\exists x A(x)}$

(22)a. Who smokes?

b. $?\exists p(p \wedge \exists x(p = A(x)))$ b′. $\boxed{A(d_1)}$ $\boxed{A(d_2)}$ $\boxed{\dots}$

In order for this account to make sense, a question meaning, e.g. the sets of propositions depicted in (21b′) or (22b′), should not be taken to represent the set of possible answers to the question, as for example in the Hamblin/Karttunen tradition, but rather as the set of propositions whose truth value is under discussion. Otherwise, for example, we would predict that question (21a) does not have a negative answer.

Interestingly, this analysis of questions allows for a perspicuous representation of the ambiguity of 'disjunctive' questions like (23), between a polar reading (expected answers: *yes*/*no*) and an alternative reading (expected answers: *coffee*/*tea*) (von Stechow 1990).

(23) Do you want coffee or tea?

a. polar reading: $?(A \vee B)$ a′. $\boxed{A \vee B}$

b. alternative reading: $?\exists p(p \wedge (p = A \vee p = B))$ b′. $\boxed{A}$ $\boxed{B}$

In example (23), intonation seems to play a disambiguating role. In alternative questions, the alternatives are normally stressed. This suggests an interesting parallelism, which deserves further investigation, between the representations in (23a) and (23b) and the possible focal structures of sentence (23) (Aloni and van Rooy 2002).

To summarize, in this section we have presented a logic of the propositional alternatives introduced by a sentence that can be motivated by the analysis of interrogative sentences. Of all the constructions looked at, only a relatively small number introduce sets of genuine alternatives, namely constructions like (19b) and (20b) which crucially contain occurrences of $\exists$ or $\vee$, which are precisely the formal representations of our FC items *any* and *or*. The following two sections propose an analysis of free choice licensing operators as quantifiers over these sets of alternatives.

4 Modals

4.1 Modals as operators over propositional alternatives

In this section I propose an analysis of modal expressions as operators over propositional alternatives. Consider the following options of modal operators applying to sequences of propositions rather than to single propositions.

(24) a. $\langle\Diamond\rangle(\phi_1, \ldots, \phi_n) := \Diamond\, \phi_1 \vee \cdots \vee \Diamond \phi_n$
b. $[\Diamond](\phi_1, \ldots, \phi_n) := \Diamond\, \phi_1 \wedge \cdots \wedge \Diamond \phi_n$
c. $\langle\Box\rangle(\phi_1, \ldots, \phi_n) := \Box\phi_1 \vee \cdots \vee \Box\phi_n$
d. $[\Box](\phi_1, \ldots, \phi_n) := \Box\phi_1 \wedge \cdots \wedge \Box\phi_n$

$\langle\Diamond\rangle/\langle\Box\rangle$ says that at least one proposition in the sequence is possible/necessary. $[\Diamond]/[\Box]$ says that all propositions in the sequence are possible/necessary. My proposal is that *may* (or *can*) should be analyzed in terms of $[\Diamond]$, and *must* in terms of $\langle\Box\rangle$. The former involves universal quantification over the set of propositional alternatives induced in its scope, the latter existential quantification.

(25) a. $\text{MAY}\ \phi \equiv [\Diamond](\text{ALT}(\phi))$
b. $\text{MUST}\ \phi \equiv \langle\Box\rangle(\text{ALT}(\phi))$

This is reflected in the following definitions.

Definition 5 (Modal expressions)

$M, w, s \models_g \text{MAY}\ \phi$ iff $\forall\alpha \in ALT(\phi)_{M,g} : \exists v \in W : wRv \;\&\; v \in \alpha$
$M, w, s \models_g \text{MUST}\ \phi$ iff $\exists\alpha \in ALT(\phi)_{M,g} : \forall v \in W : wRv \Rightarrow v \in \alpha$

MAY and MUST operate over the sets of propositional alternatives introduced in their scope. Intuitively,

(i) MAY ϕ is true in w iff *every* alternative induced by ϕ is *compatible* with the set of accessible worlds $\lambda v.wRv$;
(ii) MUST ϕ is true in w iff *at least one* alternative induced by ϕ is *entailed* by $\lambda v.wRv$.

On this account, *may* and *must* are still analyzed in terms of compatibility and entailment with respect to a relevant set of worlds. In case the embedded sentence introduces a singleton set of propositions, the predictions of this analysis do not differ from those of the standard account. When sets of genuine alternatives are introduced, however, new predictions arise, which, as we will see in the following section, improve considerably on the standard account.

4.2 Applications

This section presents a number of applications of the analysis presented above. I first discuss the case of disjunctive modal statements and show how this analysis accounts for the phenomenon of free choice disjunction in permissions and other possibility statements. I then turn to *any* and explain the difference between necessity and possibility statements with respect to the licensing of free choice indefinites in their scope.

4.2.1 'Or' in modal statements

Let us start with an example of the interaction between *or* and *may*. Consider example (26).

(26) Vincent may be in Paris or in London.

Modal verbs are treated as sentential operators which apply to the representations of the embedded sentences. In this case, there are two possible representations for the embedded disjunction. Therefore, example (26) obtains the possible analyses in (27).

(27)a. $\text{MAY}(\exists p(p \wedge (p = A \vee p = B)))$ a′. $\boxed{A}$ $\boxed{B}$

b. $\text{MAY}(A \vee B)$ b′. $\boxed{A \vee B}$

The representations in (27) express universal quantification over the sets of alternatives represented on their right. Reading (27a) is true iff each proposition in the set (27a′) is compatible with the relevant modal base. Thus, on this reading, the sentence entails that for Vincent it is both possible to be in Paris and possible to be in London.

(28) $\text{MAY}(\exists p(p \wedge (p = A \vee p = B))) \models \Diamond A \wedge \Diamond B$

On the second reading, (27b), the sentence lacks this free choice entailment because the relevant set of alternatives is now a singleton set. On reading (27b), the sentence still entails that for Vincent it is possible to be in Paris or possible to be in London, as is expected.

(29) $\text{MAY}(A \vee B) \models \Diamond A \vee \Diamond B$, but $\not\models \Diamond A \wedge \Diamond B$

Reading (27b), sometimes called 'wide scope *or*' reading, can be brought out, on a deontic interpretation of the modal,[8] by appending to the sentence the continuation: "..., but I don't know which''.

In the following example, *or* interacts with *must*.

(30) Vincent must be in Paris or in London.

a. $\textsc{must}(\exists p(p \wedge (p = A \vee p = B)))$ a′. $\boxed{A}$ $\boxed{B}$

b. $\textsc{must}(A \vee B)$ b′. $\boxed{A \vee B}$

This example as well is ambiguous between two readings. On the first reading, represented in (30a), the sentence is true iff at least one of the two propositions in the set displayed in (30a′) is entailed by the relevant modal base. Note that on this reading, which again can be brought up by appending to the sentence "..., but I don't know which,'' the sentence does not entail that for Vincent it is both necessary to be in Paris and necessary to be in London, but it has the weaker entailment that for Vincent it is necessary to be in Paris or necessary to be in London.

(31) $\textsc{must}(\exists p(p \wedge (p = A \vee p = B))) \models \Box A \vee \Box B$, but $\not\models \Box A \wedge \Box B$

The second reading of the sentence, represented in (30b), also lacks this weaker entailment. On this reading, the sentence remains unspecific as to the exact place where Vincent must be.[9]

(32) $\textsc{must}(A \vee B) \models \Box(A \vee B)$, but $\not\models \Box A \vee \Box B$

[8] On an epistemic interpretation of the modal, this second reading does not seem to be available, as has been observed by Zimmermann (2000). On this interpretation, possibility disjunctive statements always convey a free choice implication. Our semantics does not reflect this difference between epistemic and deontic modals. A pragmatic explanation, however, can be given of Zimmermann's observation using Gazdar's (1979) notion of a clausal implicature. According to Gazdar, an utterance of a possibility disjunctive statement interpreted as $\Diamond(A \vee B)$, i.e. our (27b), implicates that for each disjunct α, the speaker doesn't know whether α is true or false. This holds irrespective of whether the original sentence is interpreted epistemically or deontically. On the epistemic interpretation, the clausal implicature would then correspond to a free choice inference, somehow blurring the distinction between (27a) and (27b). On the deontic reading, no free choice effect would arise, but instead an ignorance implicature.

[9] As recognized by Zimmermann himself, his modal analysis of disjunction fails to capture this second reading of sentence (30).

It has often been observed that, on this second reading, the sentence has the free choice effect illustrated in (33) (*must* and *may* should be interpreted in a uniform way, e.g. both deontic or both epistemic).

(33)a. Vincent must be in Paris or in London. $\Rightarrow$
b. Vincent may be in Paris and Vincent may be in London.

On the present account, sentence (33a) never entails (33b). The latter, however, can be derived by the same pragmatic reasoning that derives standard ignorance implicatures of plain disjunctions. These are illustrated in the following example, where *may* should be read epistemically.

(34)a. Vincent is in Paris or in London. $\Rightarrow$
b. Vincent may be in Paris and Vincent may be in London.

The effects illustrated in (33) and (34) easily obtain from the assumption that the speaker satisfies Grice's Maxim of Quantity (Grice 1989). Example (35) illustrates the reasoning for necessity statements.

(35)a. Speaker said $\Box(A \vee B)$, rather than the more informative $\Box A$. Why?
b. Suppose Speaker had the information that $\Box A$. Then Speaker should have said so by the Quantity Maxim.
c. Therefore, Speaker has no evidence that $\Box A$ holds.
d. Under the assumption that the speaker is maximally informed, we can conclude that $\Box A$ is false.
e. Parallel reasoning for $\Box B$.
f. The fact that $\Box A$ and $\Box B$ are both false, in combination with the original sentence $\Box(A \vee B)$, implies $\Diamond A$ and $\Diamond B$.

The case of plain disjunction is simpler in that it does not need to assume that the speaker is maximally informed.

An interesting question is whether by the same pragmatic reasoning we could also have derived the free choice implications of disjunctive possibility statements.

(36)a. Vincent may be in Paris or in London. $\Rightarrow$
b. Vincent may be in Paris and Vincent may be in London.

It is easy to see, however, that we can't, at least if we assume as alternatives to $\Diamond(A \vee B)$ the natural candidates $\Diamond A$ and $\Diamond B$. To derive these effects, pragmatic accounts of free choice phenomena[10] need to assume either more complex reasonings (Kratzer and Shimoyama 2002; Aloni 2006a) or a different and less natural choice of alternatives (Schulz 2003; Aloni and van Rooij 2007).[11] This difference in complexity between the pragmatic derivation of free choice effects of necessity *versus* possibility disjunctive statements can be taken as an indication that the latter are not implicatures after all. This is

[10] Gazdar (1979) could derive free choice effects by means of his notion of a clausal implicature, but, as I mentioned in footnote 8, this worked only for epistemic sentences.

[11] Unfortunately, we do not have the space here to go into details. See also Fox (2006).

indeed the view held in this article: free choice implications of possibility disjunctive statements do have the status of a semantic entailment.

Alonso-Ovalle (2005, 2006) has recently criticized this view, observing that free choice effects disappear in downward entailing contexts, just like Quantity implicatures do. This is not only true for the case of disjunctive necessity sentences like (33a), but also for disjunctive possibility sentences like (36a). The latter observation constitutes a real challenge for the present account. An attempt of a defense is sketched in what follows.

Let us have a closer look at the predictions of the present account with respect to disjunctive possibility sentences in downward entailing contexts. We start with the case of plain negation:

(37) You may not sing or dance.
a. $\neg\textsc{may}(A \vee B)$
b. $\neg\textsc{may}(\exists p(p \wedge (p = A \vee p = B)))$

Intuitively, (37) has two readings: (i) neither singing nor dancing is possible/allowed (probably the most prominent reading of the sentence); (ii) it is not true that both singing and dancing are possible/allowed (the only reading available for *You may not sing or you may not dance*). The present account seems to make the right predictions in this case. Reading (i) is captured by (37a), involving the trivial alternative set $\{A \vee B\}$. Representation (37b), stating that not all alternatives in $\{A, B\}$ are possible/allowed, expresses reading (ii).[12]

The following example, however, is problematic for the present account (cf. ex. (13) in Alonso-Ovalle 2005):

(38) Mom, to Sandy and Leonor: 'None of you may have this cake or that ice cream.'
a. $\neg\exists x(\textsc{may}(A(x) \vee B(x)))$
b. $\neg\exists x(\textsc{may}(\exists p(p \wedge (p = A(x) \vee p = B(x)))))$

Intuitively, the sentence says that neither of the two alternatives (having the cake, having the ice cream) is possible for any of the addressees. This meaning is captured in the present account by representation (38a) involving the trivial alternative set $\{A \vee B\}$. The semantics, however, predicts the presence of reading (38b) as well, which could be paraphrased as follows: none of you is both allowed to eat the cake and allowed to eat the ice cream. Fox (2006) observes that such interpretation might be available. For example we could say (39):

(39) None of you may have this cake OR that ice cream. Everyone will be told what to eat.

[12] Note that for the parallel case involving *any*, e.g. *You may not take any card*, no ambiguity is predicted, because the weaker representation, parallel to (37b), would be ruled out by K&L's strengthening condition. For negative imperatives like *Don't sing or dance* or *Don't take any card* no ambiguity arises either, since, as we will see, negation cannot outscope the imperative operator, and negative sentences always induce trivial sets of alternatives. Again, these predictions seem to be correct.

Nevertheless, reading (38b) is definitely dispreferred. A possible way to account for this fact in the present theory and thus justify a semantic account of free choice effects, rather than a pragmatic one, would be to assume that, in pragmatic processes of disambiguation, stronger meanings are usually preferred (Chierchia 2004). This would explain not only why free choice effects disappear in downward entailing contexts—e.g. why (38a) entails (38b)—but also why free choice readings are prominent in positive environments—e.g. why (27a) entails (27b).

4.2.2 ‘Any’ in modal statements

In this section we evaluate our predictions concerning the interactions of modals and *any*. Let us start with an example of a possibility statement.

(40) Vincent may be anywhere.

On this account, the embedded existential sentence allows only two possible representations (other possibilities being logically equivalent). By applying the modal to these representations we obtain the following possible analyses for (40).

(41)a. $\text{MAY}(\exists p(p \wedge \exists x(p = A(x))))$ a′. $\boxed{A(d_1)}$ $\boxed{A(d_2)}$ $\boxed{\ldots}$

b. $\text{MAY}(\exists x A(x))$ b′. $\boxed{\exists x A(x)}$

Assuming K&L’s widening and strengthening conditions, only (41a) can serve as a representation for sentence (40), because it is only in this construction that domain widening does not lead to a loss of information. Example (41a) is true iff each of the propositions in (41a′) (that Vincent is in d_1, that Vincent is in d_2, etc.) is compatible with the relevant modal base. Representation (41a), therefore, entails the universal sentence ‘For each *d*, Vincent may be there’.

(42) $\text{MAY}(\exists p(p \wedge \exists x(p = A(x)))) \models \forall x \Diamond A(x)$

Example (41b), which quantifies over a singleton set containing an existential proposition, does not satisfy the strengthening condition; therefore, it cannot serve as representation for (40). Example (41b), however, can be used to express the non-specific reading of sentences like (43) or (44).

(43) A philosopher may come.

(44) Some philosopher may come.

Possibly, example (43) can receive both logical analyses in (41), with the universal-like interpretation (41a) expressing the generic reading of the sentence. Representation (41a), however, is certainly not available for example (44), which never yields a universal-like/generic interpretation. Note that *some* (like *any*, but unlike *a*) is picky in its distribution. For example, it cannot occur within the immediate scope of negation or as a generic. I expect that an explanation of why representation (41a) is ruled out for (44) (and maybe for (43) as well) should follow from a proper theory of the distribution of these indefinite expressions (Haspelmath 2000; Farkas 2002; Szabolcsi 2004).

Examples (43) and (44) also have a specific interpretation, which in this framework can be represented by allowing the existential quantifier to take wide scope over the modal operator, as follows:

(45)a. $\exists x(\textsc{may}(A(x)))$ b. $\boxed{A(x)}$

Sentence (45a) is true iff there is an individual *d* such that the proposition that *d* is such that *A* is consistent with the relevant modal base. Even though different alternative propositions are considered during the evaluation, the modal here always quantifies over singleton sets like (45b). The universal force of the possibility operator is then trivialized, and the sentence receives an existential interpretation. Clearly, domain widening does not strengthen the meaning here, so this representation is not available for our original *any* sentence, in accordance with our intuitions.

At last we turn to *any* in a necessity statement.

(46) #Vincent must be anywhere.

As is easy to see, we correctly predict that example (46) is out because domain widening does not lead to a stronger statement on any of its possible readings in (47).

(47) a. $\textsc{must}(\exists p(p \wedge \exists x(p = A(x))))$ a$'$. $\begin{array}{|c|}\hline A(d_1) \\ \hline A(d_2) \\ \hline \ldots \\ \hline \end{array}$

b. $\text{MUST}(\exists x A(x))$ b′. $\boxed{\exists x A(x)}$

c. $\exists x(\text{MUST}(A(x)))$ c′. $\boxed{A(x)}$

On the present analysis, *must*, in contrast with *may*, does not have the ability to change the quantificational force of an indefinite in its scope. Therefore, we seem to predict that, in necessity statements, an indefinite with *a* cannot receive a generic interpretation and *any* is not allowed. There are, however, examples of necessity statements in which *a* can be interpreted generically and *any* is licensed. Consider the following sentences:

(48) a. A car must have security belts. (generic reading available)
b. Any car must have security belts.

The (a) example can receive a generic interpretation and the (b) example is grammatical. Therefore, if their possible analyses are as in (47), they seem to constitute counterexamples to our theory. ‘Having security belts’ is an example of a so-called ‘individual-level’ predicate. Individual-level predicates have been argued to be inherently generic; that is, they are required to occur in the scope of a generic operator in order to be felicitous (Chierchia 1995). A possible solution for (48) would then be that it is the generic operator, and not *must*, which allows a generic interpretation of *a* and licenses *any* in these examples. The sentences in (48) would then be analyzed as follows:

(49) $\text{MUST}(\text{GEN}_x\ (\exists x Cx; HSBx))$

This analysis is confirmed by the fact that if we leave out *must* from the sentences in (48), nothing changes with respect to their licensing universal-like interpretations.

(50) a. A car has security belts. (generic reading available)
b. Any car has security belts.

Example (50a) can have a generic interpretation and (50b) is grammatical. Their analysis in (51) accounts for these facts:

(51) $\text{GEN}_x(\exists x Cx; HSBx)$

There is, however, a loose end that I should attend to before closing this section. Consider the following pair. Example (52a) is from Heim (1982).

(52) a. A car must be parked in the garage. (generic reading available)
b. (?) Any car must be parked in the garage.

Example (52a) can be interpreted generically and (52b) is acceptable (at least to some speakers). Note that the solution described for (48) is not available here. If we assumed the analysis in (49) for the sentences in (52), then we would make the wrong predictions about the following facts:

(53) a. A car is parked in the garage. (*no* generic reading available)
b. *Any car is parked in the garage.

Interestingly, *must* in examples (52a) (on its generic reading) and (52b) can only be interpreted deontically (Kratzer 1995), whereas (48a) (on its generic reading) and (48b) also allowed an epistemic interpretation. I am not sure how we should account for these facts. The universal effect of *a* and *any* in (52) is the result of binding by an operator with universal force. If we want to maintain my analysis of *must* as an 'existential' quantifier, we will have to assume the presence of another operator here, e.g. a generic operator as above. However, we need evidence for this assumption. As we know, the predicate 'being parked in the garage' is stage-level, and stage-level predicates do not require generic operators. A possible solution might be to assume that deontic *must*, but crucially not epistemic *must*, has the ability, under specific circumstances, to transform a stage-level, predicate into an individual-level predicate. The examples in (52) could then receive roughly the following analysis, which would account for their possible universal-like interpretations (and would not contrast with the facts in (53)):[13]

(54) $\textsc{gen}_x(\exists x Cx;\ \textsc{must}(PGx))$

A possible explanation for why this analysis does not support an epistemic interpretation could be that, as has been sometimes argued, epistemic *must* cannot occur in such an embedded position (Brennan 1993; von Fintel and Iatridou 2003).

To summarize, in this section we have presented an analysis of possibility and necessity modals as operators over sets of propositional alternatives. We have next shown how the analysis (i) accounts for the different readings of disjunctive modal statements and their free choice implications; and (ii) solves K&L's problems in explaining the difference between possibility and necessity operators with respect to licensing FC *any* in their scope. The following section extends the proposal to imperative sentences.

5 Imperatives

As already mentioned in the introduction, *any* and *or* in imperatives give rise to free choice effects. These are illustrated in (55):

(55) a. Do any x! $\Rightarrow$ For all x: you may do x!
b. Do x or y! $\Rightarrow$ You may do x and you may do y!

Following the classical literature on the subject I call imperatives like (55a, b) *choice-offering* imperatives. As an illustration of their free choice implications, consider the following two examples, which are modified versions of examples from Hamblin (1987) and Mastop (2005) respectively.

[13] The sentences in (52) seem to quantify over cars which must be parked rather than over all cars. This type of 'topical' domain restriction is disregarded in representation (54).

(56) GRANDMA: Take any card!
(Kid gets up to pick a card.)
GRANDMA: ??? Don't you dare take the ace!

(57) MOTHER: Do your homework or help your father in the kitchen!
(Son goes to the kitchen.)
FATHER: Do your homework!
SON: But, Mom told me I could also help you in the kitchen!

The most natural interpretation of Grandma's and Mother's imperatives in (56) and (57) is as one presenting a choice between different actions. Grandma's subsequent imperative in (56) and Father's imperative in (57) negate or restrict this freedom of choice. Therefore, the latter is rejected, and the former strikes us as out of place.

In the classical literature on this subject, it has often been observed that beyond the choice-offering interpretation illustrated by (57), disjunctive imperatives also allow a weaker *alternative-presenting* interpretation (Åquist 1965). Under this interpretation, the implication in (5b) is not warranted and Ross's paradox discussed in the introduction does not arise. Alternative-presenting interpretations are definitely marginal. The following dialogue from Rescher and Robison (1964) has been suggested as a possible illustration.

(58) TEACHER: John, stop that foolishness or leave the room!
(John gets up and starts to leave.)
TEACHER: Don't you dare leave this room!

The authors comment on the example as follows: "Here Teacher's second order is neither incompatible with the first nor an abrogation of it, but gives a clarification by excluding one of the initial alternatives" (Rescher and Robison 1964, p. 179, fn. 1).

In what follows I will offer an analysis of imperatives in the framework presented in the previous sections and show how it allows us to capture, first, the contrast between choice-offering and alternative-presenting disjunctive commands and, second, the meaning and distribution of *any* in imperative constructions.

5.1 Imperatives as operators over alternatives

The logic of alternatives presented in Sect. 3 supplies us with a straightforward method to characterize the compliance conditions of imperative $!\phi$, namely by identifying them with the set of alternatives induced by ϕ.

(59) Compliance Conditions($!\phi$) $= ALT(\phi)$

As is easy to see, this characterization gives us a natural account of the peculiarity of choice-offering imperatives. The compliance conditions of no-choice imperatives are singleton sets of propositions. For example, the compliance conditions of 'Post this letter!' consist of the set containing the proposition ‘that the hearer posts the letter’. Choice-offering imperatives, instead, crucially involve genuine sets of propositional alternatives. For example, the compliance conditions of ‘Post this letter or burn it!’, on its choice-offering reading, will contain the two propositions ‘that the hearer posts the letter’ and ‘that the hearer burns the letter’. Those of ‘Take any card!’ will include ‘that the hearer takes the ace of hearts’, ‘that the hearer takes the king of spades’, ... Each of these propositions represents a possible way to comply with the command (or request, advice, etc.) expressed by the imperative.

Strictly speaking, imperatives lack truth conditions. This would suggest that we identify their meaning with their compliance conditions. There is a sense, however, in which the utterance of an imperative expresses some fact about the desire state of the speaker. In order to account for this intuition, in this article I shall assume that imperatives $!\phi$ denote propositions that specify desirable situations.[14] This means that they are interpreted with respect to a modal base $\lambda v.wRv$ expressing the desires of (one of) the participants to the conversation at world w. This approach will supply us with a notion of entailment which uniformly applies to indicative and imperative sentences.

I propose the following analysis for imperatives $!\phi$, where ‘!’ is an operator over the set of propositional alternatives introduced in its scope.

Definition 6 (Imperatives)

$$M, w, s \models_g !\phi \quad \text{iff} \quad \forall\, \alpha \in ALT(\phi)_M, g : \exists\, v \in W : wRv \;\&\; v \in \alpha \;\& \\ \forall v \in W : \exists\, \alpha \in ALT(\phi)_M, g : wRv \Rightarrow v \in \alpha$$

Imperative $!\phi$ is true in w iff

(i) *every* alternative induced by ϕ is *compatible* with the desire state $\lambda v.wRv$;
(ii) the union of all these alternatives is *entailed* by $\lambda v.wRv$.

Intuitively, clause (ii) expresses the fact that if I say ‘Post the letter or burn it!’ then, in each of my desirable worlds, it should hold that either the letter is posted or it is burnt. Clause (i) expresses the fact that, in this case, my desires must be consistent with both options.

On my account, the modal force of an imperative is then identified with that of the so-called ∇ operator, which is ordinarily defined as follows:

(60) $\nabla(\phi_1, \ldots, \phi_n) := \Diamond\, \phi_1 \wedge \ldots \Diamond\, \phi_n \wedge \Box(\phi_1 \vee \ldots \vee \phi_n)$

The ∇ operator applies to a sequence of propositions and states that each of these is possible and their union is necessary. One interesting characteristic of the ∇ operator is that both possibility and necessity can be defined in terms of it:

(61) a. $\Diamond\, \phi := \nabla(\phi, T)$
b. $\Box\, \phi := \nabla(\phi) \vee \nabla\emptyset$ ($\equiv \nabla(\phi)$ in serial frames)

[14] Of course something has to be said with respect to the non-assertive behavior of imperatives. Cf. Schwager (2005) for a recent attempt.

It has often been observed that imperatives come with a puzzling range of illocutionary potential: commands ($\Box$), warnings, ... but also *permissions* ($\Diamond$) (Schwager 2005). The double nature of the ∇ operator might be what is needed to reconcile necessity and possibility usages of imperative sentences.

5.2 Applications

This section discusses a number of applications of the analysis presented above. I first present the case of disjunctive imperatives and see how the ambiguity between choice-offering and alternative-presenting *or* is captured by this analysis. I then turn to explain the distribution and meaning of *any* in imperative sentences.

5.2.1 'Or' in imperatives

On the present account, disjunctive imperatives like (62) are predicted to be ambiguous between a choice-offering reading, represented in (62a), and an alternative-presenting reading, represented in (62b).

(62) Do *a* or *b*!

a. Choice-offering: $!\exists p(p \wedge (p = A \vee p = B))$

a′. $\boxed{A}$ $\boxed{B}$

b. Alternative-presenting: $!(A \vee B)$

b′. $\boxed{A \vee B}$

The choice-offering reading in (62a) introduces the set containing the two propositions 'that the hearer does *a*' and 'that the hearer does *b*', both expressing a possible way of complying with the command expressed by the imperative. The weaker alternative-presenting reading in (62b), instead, induces the singleton set containing the disjunctive proposition 'that the hearer does *a* or *b*'. Given clause (ii) of our definition, both readings entail that the hearer must do *a* or *b*. However, since by clause (i), all the alternatives induced by the embedded clause must be consistent with the modal base, on the first choice-offering reading, but not on the alternative-presenting one, the sentence entails that for the hearer it is both allowed to do *a* and allowed to do *b*. A further related consequence is that only on the alternative-presenting reading is the sentence derivable from 'Do a!' and compatible with 'Don't do b!'. In (63) and (64) we summarize the predictions of the analysis with respect to this example.

Choice-offering disjunctive imperatives

(63) a. Post this letter or burn it! $\Rightarrow$ You must post this letter or burn it. You may post this letter and you may burn it.
$!\exists p(p \wedge (p = A \vee p = B)) \models \Box(A \vee B),\ \Diamond A \wedge \Diamond B$
b. Post this letter! $\not\Rightarrow$ Post this letter or burn it!
$!A \not\models\ !\exists p(p \wedge (p = A \vee p = B))$
c. Post this letter or burn it! # Don't you dare burn this letter!
$!\exists p(p \wedge (p = A \vee p = B)), !\neg B \models \Diamond B \wedge \neg \Diamond B$

Alternative-presenting disjunctive imperatives

(64) a. Stop that foolishness or leave the room! $\not\Rightarrow$ You may stop that foolishness and you may leave the room.
$!(A \vee B) \not\models \Diamond A \wedge \Diamond B$
b. Stop that foolishness! $\Rightarrow$ Stop that foolishness or leave the room!
$!A \models\ !(A \vee B)$
c. Stop that foolishness or leave the room! Don't you dare leave this room!
$!(A \vee B), !\neg B \models \neg \Diamond B$, but $\not\models \Diamond B$

As observed above, alternative-presenting readings are marginal. Again this could be explained by a pragmatic preference for stronger interpretations (cf. the discussion at the end of Sect. 4.2.1). Free choice interpretations always entail their alternative-presenting counterparts.

One last point should be addressed before turning to *any*-imperatives. Consider the following example:

(65) To pass the seminar, write a paper, give a presentation, or take an oral exam.

Assume that in actuality one only gets credit for an oral exam (obligatory) combined with either giving a presentation or writing a paper. As an anonymous reviewer observed, (65) would be misleading in such a scenario. However, according to the imperative semantics proposed, the sentence would be true. As a solution to this problem, we might assume that imperatives do not quantify over sets of Hamblin alternatives like $\{A, B, C\}$, but rather over sets of mutually exclusive propositions like $\{A \wedge \neg(B \vee C), B \wedge \neg(A \vee C), C \wedge \neg(A \vee B)\}$ (Menéndez-Benito 2005). Example (65), still interpreted in terms of the ∇ operator, would entail, for example, that writing a paper would be enough to pass the seminar, and, therefore, (65) would be false in the given scenario. Menéndez-Benito (2005) showed that the assumption of mutually exclusive alternatives would also overcome similar difficulties arising for possibility *any*-sentences. Consider the following scenario from her dissertation:

(66) One of the rules of the card game Canasta is: when a player has two cards that match the top card of the discard pile, she has two options: (i) she can take all the cards in the discard pile or (ii) she can take no card from the discard pile (but take the top card of the regular pile instead).

In this scenario, the following sentence is judged false.

(67) In Canasta, you can take any of the cards from the discard pile when you have two cards that match its top card.

My analysis, however, would predict (67) to be true in (66). This problem again could be solved by assuming that MAY quantifies over sets of mutually exclusive alternatives like {you take only card a, you take only card b, ...}, rather than sets of overlapping propositions like {you take card a, you take card b, ...}. Sentence (67) would then be false in (66), because, for example, it would entail that taking exactly one card from the discard pile is allowed, contradicting the rules of the game.

Although an account in terms of mutually exclusive propositions is very promising, in particular in how it extends to explain the distribution of *any* in episodic and modal sentences (cf. Menéndez-Benito 2005, for details), an alternative solution to these problems, compatible with the present analysis, could derive the exclusiveness condition by pragmatic means, as a standard scalar implicature of disjunction or indefinite phrases, rather than by syntactic/semantic encoding as proposed in Menéndez-Benito (2005). On a pragmatic account, sentences (65) and (67) are judged unacceptable in the described situations, not because they are false but because they are pragmatically odd, just like an utterance of '*A* or *B*' would be in a situation in which both *A* and *B* are true, or an utterance of 'a/some *P* is *Q*' would be in a situation in which all *P* are *Q*.

5.2.2 'Any' in imperatives

We turn now to the analysis of *any* in imperatives. Example (68a) is analyzed as in (68b), which induces the set containing the propositions 'that the hearer takes the ace of hearts', 'that the hearer takes the king of spades', etc. Again each of these propositions intuitively represents a possible way to comply with the command (or request, advice, etc.) expressed by the imperative.

(68) a. Take any card!

b. $!\exists p(p \wedge \exists x(p = A(x)))$

b′.

$A(d_1)$
$A(d_2)$
$\ldots$

Given clause (ii) of our analysis of the imperative operator, the sentence entails that the hearer must take at least one card. Clause (i) implies that each card must be a possible option.

(69) $!\exists p(p \wedge \exists x(p = A(x))) \models \Box\exists xA, \forall x\Diamond A$ (choice-offering)

Compare (68) with the following two examples, where no choice is being offered:

(70) a. Take every card!
b. $!\forall x A(x)$ b′. $\boxed{\forall x A(x)}$

(71) a. Take a card!
b. $!\exists x A(x)$ b′. $\boxed{\exists x A(x)}$

In principle our semantics predicts (71b) as a second possible reading for the *any*-imperative (68a). Intuitively, however, (68a) never obtains such a ‘purely existential’ interpretation. As we saw in example (56), imperative ‘Don’t (you dare) take the ace!’ would not be acceptable after (68a). Our representation (68b) accounts for this fact because, as we saw in (69), it entails that each card, even the ace, may be taken. Representation (71b), by contrast, lacks this entailment.

(72) $!\exists x A(x)) \models \Box\exists x A$, but $\not\models \forall x \Diamond A$ (purely existential)

In order to explain why reading (71b) is not available for sentence (68a), we can again use Kadmon and Landman’s (1993) analysis. As we saw, according to their account, *any*-phrases are indefinites which induce maximal *widening* of the domain as part of their lexical meaning. *Any* is licensed only in contexts where domain widening leads to *strengthening* of the statement made. Going back to our examples, in the ‘purely existential’ reading (71b), widening the domain would weaken the statement. This explains why this reading is not available for the *any*-sentence (68a). But what about the choice-offering reading in (68b)? Why is this representation available? Unfortunately, domain widening in this case does not make our statement stronger. However, it does not weaken the statement either. None of the wide or the narrow interpretations of sentence (68b) entail each other. Domain widening in this context creates a new meaning. This, I would like to suggest, supplies enough reason for widening to occur.

6 Conclusion and further research

I have proposed an analysis of *may*, *must*, and imperatives as operators over sets of propositional alternatives. This gave us an account of their sensitivity to the alternatives introduced by free choice *any* and *or* in their scope. The interpretation of *may* involved a universal quantification over alternatives α which took wide scope over an existential quantification over possible worlds w ($\forall\alpha\exists w$). The evaluation of *must* combined existential quantification over alternatives with universal quantification over worlds ($\exists\alpha\forall w$). The imperative operator ‘!’ corresponded to the following combination: $\forall\alpha\exists w$ & $\forall w\exists\alpha$. It is tempting to extend this analysis to other sentential operators. If we follow this line, all free choice licensing operators could then be treated as universal quantifiers ranging over sets of propositional alternatives. For example, the generic operator, GEN, would involve *universal* quantification over both alternatives and worlds ($\forall\alpha\forall w$). Possibility adverbs like *maybe* or *perhaps* would instead be examples of expressions involving *existential* quantification over alternatives and worlds ($\exists\alpha\exists w$). This is supported by the fact that they do not license *any* in their scope.

(73) #Maybe/Perhaps anyone comes.

An interesting question is whether an analysis along these lines of embedding verbs like *want*, *believe,* or *know*, beyond explaining their (in)ability to license free choice items, could shed light on some of their other linguistic properties, e.g. locality effects (Butler 2003). Other open issues that deserve further investigation include subtrigging effects (Dayal 1998), the possibility of free choice readings in wide scope disjunction (Zimmermann 2000), *any* in comparatives, and the relation between modals and the generic operator.

Lastly I would like to mention one observation which originally motivated my interest in free choice phenomena. The observation concerns the relation between free choice readings of *any* and *or* and an apparent breakdown of exhaustivity as we have in so-called *mention-some* interpretations of questions. Questions normally obtain exhaustive interpretations. Question (74a) can only be completely answered by giving an exhaustive list of the invited persons.

(74) a. Who did John invite?
b. Bill. ($\Leftrightarrow$ Bill and nobody else)

Sometimes, however, a wh-question can be completely answered by mentioning just one of the positive cases. A famous example, due to Groenendijk and Stokhof, is the following, where (75b) seems to completely resolve question (75a), but still does not imply the exhaustive answer.

(75) a. Where can I buy an Italian newspaper?
b. At the station. ($\not\Leftrightarrow$ At the station and nowhere else)

Questions allowing for a mention-some interpretation are typically free choice licensing contexts. Contrast the following two examples.

(76) a. Where can I buy an Italian newspaper?
b. You can buy an Italian newspaper at the station or at the market. $\Leftrightarrow$ You can buy an Italian newspaper at the station and you can buy an Italian newspaper at the market.
c. You can buy an Italian newspaper anywhere.

(77) a. Who did John invite?
b. John invited Bill or Mary. $\not\Leftrightarrow$ John invited Bill and John invited Mary.
c. *John invited anybody.

We might formulate the hypothesis that an interrogative ϕ? can have a mention-some reading only if ϕ is a free choice licensing context. If this hypothesis is confirmed, I expect my analysis of free choice to be able to shed some new light on the *mention-some/mention-all* contrast and, eventually, contribute to an account of the phenomena discussed in this final paragraph.

Appendix

In this appendix, I spell out the derivations of the alternative sets generated by the following sentences:

(78) $\exists p(p \wedge (p = a \vee p = b))$

(79) $\exists p(p \wedge \exists x(p = Rx))$

We start with (78). The derivation for the existential case (79) proceeds in a parallel fashion.

Disjunction. Since $n(\exists p(p \wedge (p = a \vee p = b)) = 1$, by Definition 4, $\textsc{alt}(\exists p(p \wedge (p = a \vee p = b))_{M,g}$ is equivalent to the following set:

(80) $\{\{w \in W \mid M, w, q \models_g \exists p(p \wedge (p = a \vee p = b))\} \mid q \in P\} \setminus \emptyset$

By the clause for the propositional existential quantifier:

$$M, w, q \models_g \exists p(p \wedge (p = a \vee p = b)) \text{ iff } M, w, \emptyset \models_{g[p/q]} (p \wedge (p = a \vee p = b))$$

By the clauses for propositional variables, identity and disjunction this holds iff

$$w \in q \;\&\; q = g(a) \text{ or } q = g(b)$$

But then, for $q = g(a)$, $M, w, q \models_g \exists p(p \wedge (p = a \vee p = b))$ iff $w \in g(a)$. This means:

(81) $\{w \mid M, w, g(a) \models_g \exists p(p \wedge (p = a \vee p = b))\} = g(a)$

By the same reasoning:

(82) $\{w \mid M, w, g(b) \models_g \exists p(p \wedge (p = a \vee p = b))\} = g(b)$

Since for $q \neq g(a)$ and $q \neq g(b)$, it holds that $\{w \mid M, w, q \models_g \exists p(p \wedge (p = a \vee p = b))\} = \emptyset$, we can conclude:

(83) $\textsc{alt}(\exists p(p \wedge (p = a \vee p = b))_{M,g} = \{g(a), g(b)\} \setminus \emptyset$

Existential sentence. Since $n(\exists p(p \wedge \exists x(p = Rx))) = 1$, by Definition 4, $\textsc{alt}(\exists p(p \wedge \exists x(p = Rx))_{M,g}$ is equivalent to the following set:

(84) $\{\{w \in W \mid M, w, q \models_g \exists p(p \wedge \exists x(p = Rx))\} \mid q \in P\} \setminus \emptyset$

By the clause for the propositional existential quantifier:

$$M,\, w,\, q \models_g \exists p(p \wedge \exists x(p = Rx)) \;\text{ iff }\; M, w, \emptyset \models_{g[p/q]} (p \wedge \exists x(p = Rx))$$

By the clauses for propositional variables, identity, the individual existential quantifier and predication this holds iff

$$w \in q \;\&\; \exists d \in D : q = \{v \mid d \in I(R)(v)\}$$

I will write $[\![Rx]\!]_{M,g[x/d]}$ for $\{v \mid d \in I(R)(v)\}$. But then, for all d, for $q = [\![Rx]\!]_{M,g[x/d]}$, $M, w, q \models_g \exists p(p \wedge \exists x(p = Rx))$ iff $w \in [\![Rx]\!]_{M,g[x/d]}$. This means, then, for all d:

(85) $\{w \mid M, w, [\![Rx]\!]_{M,g[x/d]} \models_g \exists p(p \wedge \exists x(p = Rx))\} = [\![Rx]\!]_{M,g[x/d]}$

If for no d, $q = [\![Rx]\!]_{M,g[x/d]}$, then it holds that $\{w \mid M, w, q \models_g \exists p(p \wedge \exists x(p = Rx))\} = \emptyset$. Therefore we can conclude:

(86) $\textsc{alt}(\exists p(p \wedge \exists x(p = Rx)))_{M,g} = \{[\![Rx]\!]_{M,g[x/d]} \mid d \in D\} \setminus \emptyset$

References

Aloni, M. (2003a). Free choice in modal contexts. In M. Weisgerber (Ed.), *Proceedings of "SuB 7—Sinn und Bedeutung"* (pp. 25–37). University of Konstanz.

Aloni, M. (2003b). On choice-offering imperatives. In P. Dekker & R. van Rooy (Eds.), *Proceedings of the 14th Amsterdam Colloquium* (pp. 57–72). ILLC–University of Amsterdam.

Aloni, M. (2006a). Expressing ignorance or indifference. Modal implicatures in BiOT. To appear in B. ten Cate & H. Zeevat (Eds.), *Proceedings of the sixth international Tbilisi symposium on language, logic and computation* (LNCS). Berlin: Springer.

Aloni, M. (2006b). Free choice and exhaustification: An account of subtrigging effects. Talk presented at *Sinn und Bedeutung 11*, September 22, 2006, Barcelona, Spain.

Aloni, M., & van Rooij, R. (2002). The dynamics of questions and focus. In B. Jackson (Ed.), *Proceedings of SALT 12*. Ithaca, N.Y.: CLC Publications.

Aloni, M., & van Rooij, R. (2007). Free choice items and alternatives. In G. Bouma, I. Krämer, & J. Zwarts (Eds.), *Cognitive foundations of interpretation* (pp. 5–25). Amsterdam: Edita KNAW.

Alonso-Ovalle, L. (2005). Distributing the disjuncts over the modal space. In L. Bateman & C. Ussery (Eds.), *Proceedings of the North East Linguistics Society 35*. Amherst, Mass.: GLSA.

Alonso-Ovalle, L. (2006). Disjunction in alternative semantics. PhD dissertation, University of Massachusetts at Amherst.

Åquist, L. (1965). Choice-offering and alternative-presenting disjunctive commands. *Analysis, 25*, 182–184.

Brennan, G. (1993). Root and epistemic modal auxiliary verbs. PhD dissertation, University of Massachusetts at Amherst.

Butler, A. (2003). Predicate logic with barriers and its locality effects. In M. Weisgerber (Ed.), *Proceedings of "SuB 7—Sinn und Bedeutung"* (pp. 70–80). University of Konstanz.

Chierchia, G. (1995). Individual-level predicates as inherent generics. In G. N. Carlson & F. J. Pelletier (Eds.), *The generic book* (pp. 176–223). Chicago: The University of Chicago Press.

Chierchia, G. (2004). Scalar implicatures, polarity phenomena and the syntax/pragmatics interface. In A. Belletti (Ed.), *Structures and beyond*. Oxford: Oxford University Press.

Dayal, V. (1998). *Any* as inherently modal. *Linguistics and Philosophy, 21*, 433–476.

Dekker, P. (1993). Transsentential meditations. Ups and downs in dynamic semantics. PhD dissertation, University of Amsterdam.
Dekker, P. (2002). Meaning and use of indefinite expressions. *Journal of Logic, Language and Information, 11*, 141–194.
Farkas, D. (2002). Varieties of indefinites. In B. Jackson (Ed.), *Proceedings of SALT 12*. Ithaca, N.Y.: CLC Publications.
Fine, K. (1970). Propositional quantifiers in modal logic. *Theoria, 36*, 336–346.
von Fintel, K., & Iatridou, S. (2003). Epistemic containment. *Linguistic Inquiry, 34*(2), 173–198.
Fox, D. (2006). Free choice disjunction and the theory of scalar implicatures. Available at http://mit.edu/linguistics/www/fox/free_choice.pdf
Gazdar, G. (1979). *Pragmatics. Implicature, presupposition, and logical form*. New York: Academic Press.
Geurts, B. (2005). Entertaining alternatives: Disjunctions as modals. *Natural Language Semantics, 13*(4), 383–410.
Giannakidou, A. (2001). The meaning of free choice. *Linguistics and Philosophy, 24*, 659–735.
Grice, P. (1989), *Studies in the way of words*. Cambridge, Mass.: Harvard University Press.
Groenendijk, J., & Stokhof, M. (1984). Studies on the semantics of questions and the pragmatics of answers. PhD dissertation, University of Amsterdam.
Hamblin, C. L. (1973). Questions in Montague English. *Foundation of Language, 10*, 41–53.
Hamblin, C. L. (1987). *Imperatives*. Oxford: Basil Blackwell.
Haspelmath, M. (2000). *Indefinite pronouns*. Oxford: Oxford University Press.
Heim, I. (1982). The semantics of definite and indefinite noun phrases. PhD dissertation, University of Massachusetts at Amherst. Published in 1989 by Garland, New York.
Horn, L. (1972). On the semantic properties of logical operators in English. PhD dissertation, UCLA.
Kadmon, N., & Landman, F. (1993). Any. *Linguistics and Philosophy, 16*, 353–422.
Kamp, H. (1973). Free choice permission. *Proceedings of the Aristotelian Society, 74*, 57–74.
Karttunen, L. (1977). Syntax and semantics of questions. *Linguistics and Philosophy, 1*, 3–44.
Kratzer, A. (1977). What 'must' and 'can' must and can mean. *Linguistics and Philosophy, 1*, 337–355.
Kratzer, A. (1995). Stage-level and individual-level predicates. In G. N. Carlson & F. J. Pelletier (Eds.), *The generic book* (pp. 125–175). Chicago: The University of Chicago Press.
Kratzer, A., & Shimoyama, J. (2002). Indeterminate pronouns: The view from Japanese. In Y. Otsu (Ed.), *The proceedings of the third Tokyo conference on psycholinguistics* (pp. 1–25). Tokyo: Hituzi Syobo.
Mastop, R. (2005). What can you do? Imperative mood in semantic theory. PhD dissertation, University of Amsterdam.
Menéndez-Benito, P. (2005). The grammar of choice. PhD dissertation, University of Massachusetts at Amherst. To be published by GLSA, University of Massachusetts, Amherst.
Portner, P. (2004). The semantics of imperatives within a theory of clause types. In K. Watanabe & R. B. Young (Eds.), *Proceedings of semantics and linguistic theory 14*. Ithaca, N.Y.: CLC Publications.
Rescher, N., & Robison, J. (1964). Can one infer commands from commands. *Analysis, 24*, 176–179.
Rooth, M. (1992). A theory of focus interpretation. *Natural Language Semantics, 1*, 75–116.
Ross, A. (1941). Imperatives and logic. *Theoria, 7*, 53–71.
Schulz, K. (2003). You may read it now or later. A case study on the paradox of free choice permission. Master's thesis, University of Amsterdam.
Schwager, M. (2005). Interpreting Imperatives. PhD dissertation, Goethe-Universität, Frankfurt am Main.
Simons, M. (2005). Dividing things up: The semantics of *or* and the modal/*or* interaction. *Natural Language Semantics, 13*(3), 271–316.
von Stechow, A. (1990). Focusing and backgrounding operators. In W. Abraham (Ed.), *Discourse particles*, 6 (pp. 37–84). Amsterdam: John Benjamins.
Szabolcsi, A. (2004). Positive polarity – negative polarity. *Natural Language and Linguistic Theory, 22*(2), 409–452.
Westerståhl, D. (1984). Determiners and context sets. In J. van Benthem & A. ter Meulen (Eds.), *Generalized quantifiers in natural language* (pp. 45–71). Dordrecht: Foris.
Zimmermann, E. (2000). Free choice disjunction and epistemic possibility. *Natural Language Semantics, 8*, 255–290.