On choice-offering imperatives

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1 Introduction

The law of propositional logic that states the deducibility of *either A or B* from *A* is not valid for imperatives (Ross's paradox, cf. [9]). The command (or request, advice, etc.) in (1a) does not imply (1a) (unless it is taken in its *alternative-presenting* sense), otherwise when told the former, I would be justified in burning the letter rather then posting it.

(1) a. Post this letter! $\not\Rightarrow$ b. Post this letter or burn it!

Intuitively the most natural interpretation of the second imperative is as one presenting a choice between two actions. Following [2] (and [6]) I call these *choice-offering* imperatives. Another example of a choice-offering imperative is (2) with an occurence of Free Choice 'any' which, interestingly, is licensed in this context.

(2) Take any card!

Like (1a), this imperative should be interpreted as carrying with it a permission that explicates the fact that a choice is being offered.

Possibility statements behave similarly (see [8]). Sentence (3b) has a reading under which it cannot be deduced from (3a), and 'any' is licensed in (4).

(3) a. You may post this letter. $\not\Rightarrow$ b. You may post this letter or burn it.

(4) You may take any card.

In [1] I presented an analysis of modal expressions which explains the phenomena in (3) and (4). That analysis maintains a standard treatment of 'or' as logical disjunction (contra [11]) and a Kadmon & Landman style analysis of 'any' as existential quantifier (contra [3] and [4]) assuming, however, an independently motivated 'Hamblin analysis' for $\vee$ and $\exists$ as introducing sets of alternative propositions. Modal expressions are treated as operators over sets of propositional alternatives. In this way, since their interpretation can depend on the alternatives introduced by 'or' ($\vee$) or 'any' ($\exists$) in their scope, we can account for the free choice effect which arises in sentences like (3b) or (4). In this article I would like to extend this analysis to imperatives. The resulting theory will allow a unified account of the phenomena in (1)-(4). We will start by presenting our 'alternative' analysis for indefinites and disjunction.

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2 Indefinites and disjunction

Indefinites (e.g. 'any') and disjunction (e.g. 'or') have a common character reflected by their formal counterparts $\exists$ and $\vee$. Existential sentences and logical disjunctions assert that at least one element of a larger set of propositions is true, but not which one. Both constructions can be thought of as introducing a set of alternative propositions and, indirectly, raising the question about which of these alternatives is true. In what follows I propose a formal account of the sets of propositional alternatives introduced by indefinites and 'or' (cf. [1]).

I recursively define a function $[\bullet]_{M,g}$ where M is a pair consisting of a set of individuals D and a set of worlds W, and g is an assignment function. Function $[\bullet]_{M,g}$ maps formulae ϕ to sets of pairs $\langle w, s\rangle$ consisting of a world $w \in W$ and a sequence of values s, where the length of s is equivalent to the number $n(\phi)$ of surface existential quantifiers in ϕ, – for atoms and negations, $n(\phi) = 0$; for $\phi = \exists x\psi$, $n(\phi) = 1 + n(\psi)$, and for $\phi = \psi_1 \wedge \psi_2$, $n(\phi) = n(\psi_1) + n(\psi_2)$. (By $[\![\alpha]\!]_{M,w,g}$ I refer, as standard, to the denotation of α in M, w and g.)

Definition 1

1. $[P(t_1, ..., t_n)]_{M,g} = \{\langle\langle\rangle, w\rangle \mid \langle[\![t_1]\!]_{M,w,g}, ..., [\![t_n]\!]_{M,w,g}\rangle \in [\![P]\!]_{M,w,g}\}$;
2. $[t_1 = t_2]_{M,g} = \{\langle\langle\rangle, w\rangle \mid [\![t_1]\!]_{M,w,g} = [\![t_2]\!]_{M,w,g}\}$;
3. $[\neg\phi]_{M,g} = \{\langle\langle\rangle, w\rangle \mid \neg\exists s : \langle s, w\rangle \in [\phi]_{M,g}\}$;
4. $[\exists x\phi]_{M,g} = \{\langle ds, w\rangle \mid \langle s, w\rangle \in [\phi]_{M,g[x/d]}\}$;
5. $[\phi \wedge \psi]_{M,g} = \{\langle s_1 s_2, w\rangle \mid \langle s_2, w\rangle \in [\phi]_{M,g}\ \&\ \langle s_1, w\rangle \in [\psi]_{M,g}\}$.

Disjunction $\vee$, implication $\rightarrow$ and universal quantification $\forall$ are defined as standard in terms of $\neg$, $\wedge$ and $\exists$. Truth and entailment are defined as follows.

Definition 2 [Truth and entailment]

(i) $M, w \models_g \phi$ iff $\exists s : \langle s, w\rangle \in [\phi]_{M,g}$;
(ii) $\phi \models \psi$ iff $\forall M, \forall w, \forall g : M, w \models_g \phi \Rightarrow M, w \models_g \psi$.

In this semantics, a formula is associated with a set of world-sequence pairs, rather than, as usual, with a set of worlds. This addition is essential to derive the proper set $\text{ALT}(\phi)_{M,g}$ of alternative propositions induced by formula ϕ, which is defined as follows.

Definition 3 $\text{ALT}(\phi)_{M,g} = \{\{w \mid \langle s, w\rangle \in [\phi]_{M,g}\} \mid s \in D^{n(\phi)}\}$.

For example, the set $[P(x)]_{M,g} = \{\langle\langle\rangle, w\rangle \mid [\![x]\!]_{M,w,g} \in [\![P]\!]_{M,w,g}\}$ determines the singleton set of propositions $\{$that x is $P\}$. More interestingly, the set $[\exists x P(x)]_{M,g} = \{\langle\langle d\rangle, w\rangle \mid d \in [\![P]\!]_{M,w,g}\}$ determines the set of alternatives $\{$that d_1 is P, that d_2 is P, $\ldots\}$, containing as many elements as there are possible values for the quantified variable x.

On this account, the propositional alternatives introduced by a sentence are defined in terms of the set of possible values for an existentially quantified variable. To properly account also for the alternatives introduced by disjunctions, I propose to add to our language, variables p, q ranging over propositions, so that, for example, we can write $\exists p({}^{\vee}p \wedge\ p = {}^{\wedge}A)$ for A, where the operators ${}^{\vee}$

and $^\wedge$ receive the standard interpretation, so that, for example, $[\![^\vee p]\!]_{M,g,w} = 1$ iff $w \in g(p)$, and $[\![^\wedge A]\!]_{M,g,w} = [\![A]\!]_{M,g}$. In interaction with $\exists$ or $\vee$, this addition, otherwise harmless, extends the expressive power of our language in a non-trivial way. Although the (a) and (b) sentences below are truth conditionally equivalent, the sets of alternatives they bring about, depicted on the right column, are not the same. While the (b) representations introduce singleton sets, the (a) representations induce genuine sets of alternatives.

(5) a. $\exists p(^\vee p \wedge (p = ^\wedge A \vee p = ^\wedge B))$ a′. $\begin{array}{c}\boxed{A}\\ \boxed{B}\end{array}$

b. $\exists p(^\vee p \wedge p = ^\wedge (A \vee B))$ b′. $\boxed{A \vee B}$

(6) a. $\exists p(^\vee p \wedge \exists x(p = ^\wedge A(x)))$ a′. $\begin{array}{c}\boxed{A(d_1)}\\ \boxed{A(d_2)}\\ \boxed{\ldots}\end{array}$

b. $\exists p(^\vee p \wedge p = ^\wedge \exists x A(x))$ b′. $\boxed{\exists x A(x)}$

That these alternatives are needed is seen when we consider question semantics. If we take questions $?\phi$ to denote the sets of alternatives induced by ϕ, the pair in (5) allows us a proper representation for the ambiguity of questions like ‘Do you want coffee or tea?’ between an alternative reading (expected answers: coffee/tea), and a polar reading (expected answers: yes/no) (see [10]). The sets of alternatives induced by (6a) and (6b) can serve as denotations for constituent questions (e.g. ‘who smokes’) and polar existential questions (e.g. ‘whether anybody smokes’) respectively.

3 Imperatives

While assertion have truth conditions, imperatives have *compliance conditions*. Someone cannot be said to understand the meaning of an imperative unless he recognizes what has to be true for the command (or request, advice, etc.) issued by utterance of it to be complied with. The framework presented in the previous section supplies us with a straightforward method to characterize the compliance conditions of imperative $!\phi$, namely by identifying them with the set of alternatives induced by ϕ. For example, the compliance conditions of the imperative ‘Post this letter!’ will be the singleton set containing the proposition ‘that the addressee posts the letter’.[1] Crucially choice-offering imperatives will involve genuine sets of alternatives. For example, the compliance conditions of ‘Post this letter or burn it!’, on its choice-offering reading, will contain the two propositions: ‘that the addressee posts the letter’ and ‘that the addressee burns the letter’. Each of these propositions represents a possible way to comply with the command (or request, advice, etc.) expressed by the imperative.

Strictly speaking imperatives lack truth conditions. This would suggest to identify their meaning with their compliance conditions. There is a sense,

1. We are bypassing the fact that imperatives deal with future actions, so the relevant proposition here should be ‘that the addressee will post the letter’. See Rosja Mastop’s contribution to this volume ‘Imperatives and Tense’.

however, in which the utterance of an imperative expresses some fact about the desire state of the speaker. In order to account for this intuition, in this article, I shall assume that imperatives $!\phi$ denote propositions that specify desirable situations. This means that they are interpreted with respect to a modal base A_w expressing the desires of (one of) the participants to the conversation at world w.

Definition 4 [Imperatives] $[!\phi]_{M,g} = \{\langle\langle\rangle, w\rangle \mid \forall\alpha \in ALT(\phi)_{M,g} : \exists w' \in A_w : w' \in \alpha \;\&\; \forall w' \in A_w : \exists\alpha \in ALT(\phi)_{M,g} : w' \in \alpha\}$

On this account, !' is an operator over the set of propositional alternatives introduced in its scope. Imperative $!\phi$ is true in w iff (i) *every* alternative induced by ϕ is *compatible* with the desire state A_w; (ii) the union of all these alternatives is *entailed* by A_w. Intuitively, clause (ii) expresses the fact that if I say 'Post the letter or burn it!' then, in each of my desirable worlds, it should hold that either the letter is posted or burnt. Clause (i) expresses the fact that, in this case, my desires must be consistent with both options.

In this framework we can give a straightforward treatment of 'embedded uses' of imperatives like in 'Vincent wants you to post this letter'. We first define a relation of entailment between desire states and imperatives, as follows. State σ *entails* $!\phi$, $\sigma \models_{M,g} !\phi$ iff $\exists w : M, w \models_g !\phi$ and $A_w = \sigma$. We then assume that a sentence like 'Vincent wants ϕ!' is true in w iff Vincent's desire state in w entails $!\phi$.

Let us see now how the choice-offering imperatives discussed in the introductory part of the article are analyzed in this framework.

Applications Example (7) is ambiguous between a choice-offering reading, represented in (7a), and an alternative-presenting reading in (7b).

(7) Post this letter or burn it!

a. $!\exists p({}^{\vee}p \wedge (p =^{\wedge}A \vee p =^{\wedge}B))$ a′. $\boxed{A}$ / $\boxed{B}$

b. $!\exists p({}^{\vee}p \wedge p =^{\wedge}(A \vee B))$ b′. $\boxed{A \vee B}$

The choice-offering reading involves the set containing the two propositions: 'that the addressee posts the letter' and 'that the addressee burns the letter', both expressing a possible way of complying with the imperative. The weaker reading in (7b) instead induces the singleton set containing the proposition 'that the addressee posts the letter or burns it'. Since, by clause (i) of our definition, all the alternatives induced by the embedded clause must be consistent with the modal base, only on this second reading is the sentence compatible with a subsequent imperative: 'Do not burn the letter!' Assuming a standard treatment of $\Diamond$ and $\Box$, the following holds:

(8) a. $!\exists p({}^{\vee}p \wedge (p =^{\wedge}A \vee p =^{\wedge}B)) \models \Diamond A, \Diamond B, \Box(A \vee B)$

b. $!\exists p({}^{\vee}p \wedge p =^{\wedge}(A \vee B)) \not\models \Diamond A, \Diamond B$

Example (9a) is analyzed as in (9b) which induces the set containing the propositions 'that the addressee takes the ace of hearts', 'that the addressee takes the king of spades', ...

(9) a. Take any card!

b. $!\exists p({}^{\vee}p \wedge \exists x(p = {}^{\wedge}A(x)))$ b′. $\boxed{A(d_1)}$ $\boxed{A(d_2)}$ $\boxed{\ldots}$

Compare (9) with the following two examples where no choice is being offered:

(10) a. Take every card!

b. $!\exists p({}^{\vee}p \wedge (p = {}^{\wedge}\forall x A(x)))$ b′. $\boxed{\forall x A(x)}$

(11) a. Take a card!

b. $!\exists p({}^{\vee}p \wedge p = {}^{\wedge}\exists x A(x))$ b′. $\boxed{\exists x A(x)}$

In principle our semantics predicts (11b) as second possible reading for sentence (9a). Intuitively, however, (9a) never obtains such a 'pure' existential meaning. Imperative 'Do not take the ace!' would never be acceptable after (9a). Our representation (9b) accounts for this fact, because it entails that any card may be taken. Representation (11b), instead, lacks this entailment.

(12) a. $!\exists p({}^{\vee}p \wedge \exists x(p = {}^{\wedge}A(x))) \models \forall x \Diamond A, \Box \exists x A$

b. $!\exists p({}^{\vee}p \wedge p = {}^{\wedge}\exists x A(x)) \not\models \forall x \Diamond A$

In order to explain why reading (11b) is not available for sentence (9a), I will use Kandom and Landman's analysis of *any* (see [7]). According to their account, *any* phrases are indefinites which induce maximal *widening* of the domain as part of their lexical meaning. Crucially this widening should be for a reason, namely, they propose, the *strengthening* of the statement made. If we define the strength of an imperative in terms of entailment, $\models$, in the 'pure' existential reading (11b), widening the domain would weaken the statement. This explains why this reading is not available for the *any*-sentence (9a). But what about the 'free choice' reading in (9b)? Why is this available? Unfortunateky widening the domain in this case does not make our statement stronger. None of the wide or the narrow interpretation of sentence (9b) entail the other. We lack then an explanation of why (9a) can be interpreted at all. In order to solve this problem we have to say something more about in what sense an imperative can be said to be stronger than another.

In this framework, we have a number of alternative options for defining the relative strength of imperatives. Entailment is one possibility. The following two are other particularly interesting options.

1. $!A \models\!\approx_1 !B$ iff $\forall \alpha \in ALT(A) : \exists \beta \in ALT(B) : \alpha \subseteq \beta$;
2. $!A \models\!\approx_2 !B$ iff $\forall \beta \in ALT(B) : \exists \alpha \in ALT(A) : \alpha \subseteq \beta$.

Intuitively, imperative $!A$ is as strong$_1$ as $!B$, $!A \models\!\approx_1 !B$ iff each way of complying with $!A$ is also a way of complying with $!B$. Whereas $!A \models\!\approx_2 !B$ holds iff any way of complying with $!B$ is part of a strategy to comply with $!A$. If $!\phi \models\!\approx_1 !\psi$ and $!\phi \models\!\approx_2 !\psi$, then $!\phi \models !\psi$.

If $!A$ and $!B$ denote singleton sets, $\models\!\approx_1$ and $\models\!\approx_2$ (and $\models$) define the same notion. For example, imperative (13a) is stronger than (13b) according to both notions. Indeed, every way of satisfying (13a) satisfies (13b), and to satisfy (13b) is part of a strategy to satisfy (13a).

(13) a. Put all books in your bag! b. Put the *Tractatus* in your bag!

Once choice-offering imperatives enter the picture though, the two notions give opposite results (by $!(A \vee_c B)$ I refer to the free choice reading of a disjunctive imperative e.g. (7a)):

(14) a. Post this letter! b. Post this letter or burn it!

c. $!A \approx\!\!\!\models_1 !(A \vee_c B)$ and $!(A \vee_c B) \not\approx\!\!\!\models_1 !A$

d. $!A \not\approx\!\!\!\models_2 !(A \vee_c B)$ and $!(A \vee_c B) \approx\!\!\!\models_2 !A$

e. $!A \not\models !(A \vee_c B)$ and $!(A \vee_c B) \not\models !A$

Sentence (14a) is strictly stronger$_1$ than (14b), because posting the letter is a way to satisfy (14b), but burning the letter is not a way to satisfy (14a). On the contrary, sentence (14b) is strictly stronger$_2$ than (14a), because posting the letter is part of a strategy to satisfy (14b), but there is a way to satisfy the latter, namely burning the letter, which is not part of a strategy to satisfy (14a).

Going back to our example (9), in the 'pure' existential readings in (11b), widening the domain makes our statement weaker according to all notions $\models$, $\approx\!\!\!\models_1$ and $\approx\!\!\!\models_2$. This explains why this reading is not available for the *any*-sentence in (9). In the 'free choice' reading in (9b), widening makes the statement weaker according to notion $\approx\!\!\!\models_1$, but stronger according to notion $\approx\!\!\!\models_2$. This, I suggest, supplies enough reason for widening to occur.

References

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