Logic and conversation: neglect-zero effects at the semantics-pragmatics interface

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Introduction

- Grice's paradise: canonical divide between semantics and pragmatics
  - **Pragmatic inference**: derivable by conversational principles, cancellable, non-embeddable, ...
  - **Semantic inference**: not derivable by conversational principles, non-cancellable, embeddable, ...
- Gricean picture recently challenged by a class of inferences triggered by existential/disjunctive constructions:
  - Ignorance inference in epistemic indefinites and modified numerals
  - Free choice inferences in indefinites and disjunction
  - ....
- Common core of these inferences:
  - Although derivable by conversational principles they lack other defining properties of pragmatic inferences

  ⇒ Neither purely semantics nor purely pragmatics, rather *inferences of the 3rd kind*
- Goal: a formal account of these inferences which captures their quasi-semantic behaviour while explaining their pragmatic nature
- Strategy: develop **logics of conversation** which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction

Novel hypothesis: neglect-zero

- Inferences of the 3rd kind are
  - neither the result of conversational reasoning (as proposed in neo-gricean approaches) [$\neq$ canonical conversational implicatures]
  - nor the effect of spontaneous optional applications of grammatical operators (as in the grammatical view on FC) [$\neq$ scalar implicatures]
- Rather they are a straightforward consequence of something else speakers do in conversation, namely,
  - when interpreting a sentence they create pictures of the world and in doing so they systematically neglect structures which verify the sentence by virtue of some empty configuration (*zero-models*)
- This tendency, which I call neglect-zero, follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets (Nieder 2016, Bott et al 2019)

Novel hypothesis: neglect-zero

Illustrations

(1) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

(2) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

- Cognitive difficulty of zero-models confirmed by findings from number cognition and also explains
  - the special status of 0 among the natural numbers (Nieder, 2016)
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (Bott et al., 2019).

Beyond Gricean paradise

		pragm. derivable	cancel lable	non-embed.	proc. cost	acqui sition
Pra gma tics	Conversational implicature J is always very punctual $\rightsquigarrow$ J is not a good philosopher	+	+	+	high	late
Sem ant ics	Classical entailment I read some novels $\rightsquigarrow$ I read something	−	−	−	low	early
3rd Kind	FC disjunction You may do A *or* B $\rightsquigarrow$ You may do *A*	+	?	?	low	early
	Scalar implicature I read some novels $\rightsquigarrow$ I didn't read all novels	+	+	?	high	late

Plan of today

1. Free choice (FC) inference: mismatch between logic and language
2. FC inferences as neglect-zero effects in Bilateral State-based Modal Logic (BSML)[1]
3. Compare different implementations of neglect-zero effects in variants of BSML

[1] Aloni (2021). Logic and Conversation: the case of free choice. Available at `https://www.marialoni.org/resources/Aloni2021.pdf`.

Free choice (FC)

- FC inference:

(3) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

- Classical examples:

(4) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(5) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

The paradox of free choice

- Free choice permission in natural language:

(6) You may (A or B) $\rightsquigarrow$ You may A

- But (7) not valid in classical deontic logic:

(7) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(8)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (8) uses the following valid principle:

(9) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$ [Modal Addition]

- Natural language counterpart of (9), however, seems invalid:

(10) You may post this letter $\not\rightsquigarrow$ You may post this letter or burn it. [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice Permission:

(11)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by modal addition]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic solutions [⇒ keep logic as is]
  - FC inferences are pragmatic inferences, conversational implicatures
  - ⇒ step leading to 3 is unjustified
- Semantic solutions [⇒ change the logic]
  - FC inferences are semantic entailments
  - ⇒ step leading to 3 is justified, but step leading to 2 is no longer valid
- My proposal: a logic-based account beyond canonical semantics vs pragmatics divide
  - FC neither semantic entailments nor derived by gricean reasoning, rather consequence of pragmatic factors modeled in a *logic of conversation*: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha$, but $\Diamond(\alpha \vee \beta) \not\models \Diamond\alpha$
  - Upshot logic-based account: hybrid behaviour naturally derived

Free choice: semantics or pragmatics

Argument against semantic accounts of FC

Free choice effects systematically disappear in negative contexts:

(12) Dual Prohibition (Alonso-Ovalle 2005)

a. You are not allowed to eat the cake or the ice-cream
$\rightsquigarrow$ You are not allowed to eat either one

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

- Unexpected on a semantic account where $\Diamond(\alpha \vee \beta) \models \Diamond\alpha \wedge \Diamond\beta$
- Predicted by pragmatic accounts: pragmatic inferences do not embed under logical operators

Free choice: semantics or pragmatics

Argument against pragmatic accounts of FC

Free choice effects embeddable under universal quantification:

(13) Universal FC (Chemla 2009)

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

- Unexpected on a pragmatic account: pragmatic inferences do not embed under logical operators
- Predicted by semantic accounts where $\Diamond(\alpha \vee \beta) \models \Diamond\alpha \wedge \Diamond\beta$

Free choice: semantics or pragmatics

Argument against most accounts

Free choice effects also arise with wide scope disjunctions:

(14) Wide Scope FC (Zimmermann 2000)

a. Detectives may go by bus or they may go by boat. $\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton. $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

Free choice: summary data and predictions

(15)
a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

	N Scope FC	Dual Prohibition	Universal FC	W Scope FC
Semantic	yes	no	yes	no
Pragmatic	yes	yes	no	no

Free choice: semantics or pragmatics?

- I propose a hybrid approach where
  - FC inference derived by modelling the intrusion of pragmatic factors in the process of interpretation
- Intruding pragmatic factor: neglect-zero
  - a tendency of language users to systematically neglect zero-models when engaging in linguistic interpretations
- Neglect-zero modeled using tools from team semantics

Team-based logics

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states
- Assertion & rejection conditions are modeled rather than truth

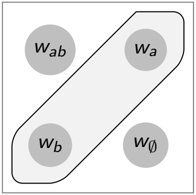
$$M, s \models \phi, \text{ ``}\phi \text{ is assertable in } s\text{''}, \quad \text{with } s \subseteq W$$
$$M, s \mathrel{=\!\!|} \phi, \text{ ``}\phi \text{ is rejectable in } s\text{''}, \quad \text{with } s \subseteq W$$

- Inferences relate speech acts, rather than propositions and therefore might diverge from semantic entailments (*reasonable inference*, Stalnaker73)

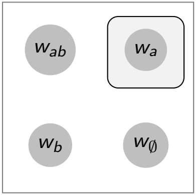
BSML: split disjunction

- s supports $\phi \vee \psi$ iff s is the union of two substates, each supporting one of the disjuncts:

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \ \& \ M, t \models \phi \ \& \ M, t' \models \psi$$



(a) $\models a \vee b$

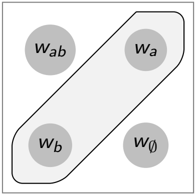


(b) $\models a \vee b$

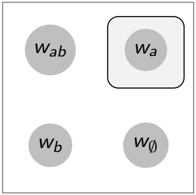
- State $\{w_a\}$ example of a **zero-model** for $a \vee b$!
  - In BSML, $\emptyset$ supports all classical formulas: split disjunction delivers classical disjunction as long as the disjuncts are classical ($s = s \cup \emptyset$);
- Core idea: Pragmatic enrichment, modelled as a neglect-zero effect, crucially rules out the possibility of the empty state acting as one of the relevant substates for evaluation

BSML: pragmatically enriched disjunction

- s supports $[\phi \vee \psi]^+$ iff s is the union of two **non-empty** substates, each supporting one of the disjuncts.



(c) $\models a \vee b$ & $\models [a \vee b]^+$



(d) $\models a \vee b$; but $\not\models [a \vee b]^+$

- Pragmatically enriched disjunctions require both disjuncts to be **live possibilities** (as in Zimmermann 2000)

(16) Wataru is in Japan or in Europe $\rightsquigarrow$ Wataru might be in Japan and might be in Europe

$\mapsto$ We derive ignorance effects of plain disjunction

- Pragmatic enrichment function $[\ ]^+$ defined in terms of NE

BSML: NE and pragmatic enrichment

- The non-emptiness atom (NE) requires the supporting state to be non-empty:

$$M, s \models \mathrm{NE} \quad \text{iff} \quad s \neq \emptyset$$

- Pragmatically enriched formulas $[\alpha]^+$ come with the requirement to satisfy NE distributed along each of their subformulas:

$$\begin{array}{rcl} [p]^+ & = & p \wedge \mathrm{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \mathrm{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \mathrm{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \mathrm{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \mathrm{NE} \end{array}$$

- Main result: in BSML pragmatic enrichment has non-trivial effect only when applied to positive disjunctions:
  - $\mapsto$ we derive FC effects (for pragmatically enriched formulas);
  - $\mapsto$ pragmatic enrichment vacuous under negation.

Pragmatic enrichment in BSML

Predictions

- By pragmatically enriching every formula, we derive:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (with restrictions)
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$
- NE-free fragment of BSML equivalent to classical modal logic:
  $\alpha \models_{BSML} \beta$ iff $\alpha \models_{CML} \beta$ (α, β are NE-free)

Resulting picture

- Empirically correct: Subtle predictions wrt wide scope FC confirmed by pilot experiment (Cremers et al., 2017);
- Cognitively plausible
  - BSML shows that FC follows from the assumption that when interpreting sentences language users neglect zero-models
  - Zero-models neglected because cognitively taxing
  - Low processing costs and early acquisition of FC explained

Modelling neglect-zero effects: different implementations

- More ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function $[\ ]^+$ defined in terms of NE
  - Model-theoretically, by ruling out $\emptyset$ from the set of possible states $\mapsto$ BSML*
- Both implementations derive:
  - $\mapsto$ FC effects (narrow and wide scope FC, the latter with restrictions);
  - $\mapsto$ cancellations of FC effects under negation (dual prohibition).
- But conceptual and empirical differences:
  - Only BSML* predicts Negative FC: $\Diamond\neg(\alpha \wedge \beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$
  - Only in BSML, where $\emptyset$ is part of the building blocks, locality and suspension of neglect-zero effects can be modeled
- NEXT: compare different implementations of neglect-zero in variants of BSML and explore whether they can be used to model different interpretation strategies language users employ in conversation

Bilateral State-Based Modal Logic (BSML)

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$

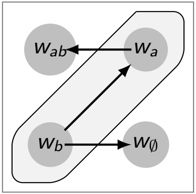
where $p \in A$.

Models and States

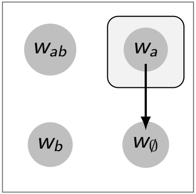
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model [$s \neq \emptyset$ in BSML*]

Examples

for $A = \{a, b\}$



(e) $\not\models a$; $\models \Diamond a$



(f) $\models a$; $\not\models \Diamond a$

Semantic clauses

$[M = \langle W, R, V \rangle; s, t, t' \subseteq W]$

$$
\begin{array}{rcl}
M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\
M, s \mathrel{=\!\!|} p & \text{iff} & \forall w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \mathrel{=\!\!|} \phi \\
M, s \mathrel{=\!\!|} \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \mathrel{=\!\!|} \phi \vee \psi & \text{iff} & M, s \mathrel{=\!\!|} \phi \ \&\ M, s \mathrel{=\!\!|} \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \mathrel{=\!\!|} \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \mathrel{=\!\!|} \phi \ \&\ M, t' \mathrel{=\!\!|} \psi \\
M, s \models \Diamond\phi & \text{iff} & \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi \\
M, s \mathrel{=\!\!|} \Diamond\phi & \text{iff} & \forall w \in s : M, R[w] \mathrel{=\!\!|} \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \mathrel{=\!\!|} \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R[w] = \{v \in W \mid wRv\}$

Box

- $\Box\phi := \neg\Diamond\neg\phi$

$$\begin{array}{lll} M, s \models \Box\phi & \text{iff} & \text{for all } w \in s : M, R[w] \models \phi \\ M, s \dashv \Box\phi & \text{iff} & \text{for all } w \in s : \text{there is a } t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \dashv \phi \end{array}$$

where $R[w] = \{v \in W \mid wRv\}$

Logical consequence

- $\phi \models \psi$ iff for all M, s : $M, s \models \phi \;\Rightarrow\; M, s \models \psi$

Pragmatic enrichment

For NE-free α, $[\alpha]^+$ defined as follows:

$$\begin{array}{rcl} [p]^+ & = & p \wedge \text{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \text{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \text{NE} \end{array}$$

Results propositional BSML

Before pragmatic intrusion

- The NE-free fragment of BSML is equivalent to classical modal logic: $\alpha \models_{BSML} \beta$ iff $\alpha \models_{CML} \beta$ (α, β are NE-free)
- But we can capture infelicity of epistemic contradictions by putting constraints on epistemic accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

After pragmatic intrusion

- FC inferences derived for pragmatically enriched disjunction:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
  - **Modal disjunction**: $[\alpha \vee \beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is state-based)
- Only disjunctions in positive environments affected by pragmatic intrusion:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$
  - Double negation: $\neg\neg\Diamond(\alpha \vee \beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - **Negative** FC: $[\neg\Box(\alpha \wedge \beta)]^+ \not\models \neg\Box\alpha \wedge \neg\Box\beta$
- Status of Modal disjunction and Negative FC is debated in the literature

Modal disjunction and Negative FC

- **Experimental research**: modal disjunction and negative FC inference exist but are less available than positive FC:

(17) Modal disjunction (Tieu et al., 2019)

a. Angie bought the boat or the car $\rightsquigarrow$ Angie might have bought the boat and might have bought the car

b. $\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(18) Negative FC (Marty et al., 2021)

a. It is not required that Mia buys apples and bananas $\rightsquigarrow$ It is not required that Mia buys apples and that Mia buys bananas

b. $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$

- BSML$^+$: BSML + global pragmatic enrichment

$$\alpha \models_{BSML^+} \beta \text{ iff } [\alpha]^+ \models_{BSML} [\beta]^+$$

- Mismatch between BSML$^+$ and experimental findings:

			BSML$^+$
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**strong**	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**weak**	-
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**weak**	+

BSML$^+$ vs BSML*

- BSML*: like BSML, but $\emptyset$ is not among the possible states;
- **Fact:** Let α, β be classical positive formulas. Then

$$\alpha \models_{BSML^*} \beta \text{ iff } [\alpha]^+ \models_{BSML} [\beta]^+$$

- But this does not hold in general. In BSML*, FC inferences generated also for negative conjunctions ($\Rightarrow$ Negative FC):

$$\Diamond\neg(\alpha \wedge \beta) \quad \models_{BSML^*} \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$
$$\neg\Box(\alpha \wedge \beta) \quad \models_{BSML^*} \quad \neg\Box\alpha \wedge \neg\Box\beta$$

- Still not a perfect match with results experimental research:

			BSML$^+$	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**s**	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	+
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**w**	+	+

- Part of the problem: BSML$^+$ and BSML* model neglect-zero effects as global phenomena, but are they?

Neglect-zero effects: why and when?

- Neglect-zero: tendency of language users to neglect zero-models in ordinary conversation
- Why? Zero-models are cognitively taxing
- But are neglect-zero effects optional or obligatory? And if optional, can they be suspended locally or only globally?

Conjectures

- Despite their cognitive cost, zero-models are not always neglected.
  - Global suspension of neglect-zero effects in logical-mathematical reasoning $\mapsto$ $\text{BSML}^{\emptyset}$
- Two kinds of neglect-zero effects:
  1. Global nz effects modeled by BSML^{*} $\mapsto$ *weak*
  2. Local nz effects triggered by certain expressions as result of lexicalisations $\mapsto$ *strong* $\mapsto$ BSML^{lex}
- Overview of labels
  - BSML: Bilateral State Based Modal Logic (Logical System)
  - BSML^{*}: BSML without $\emptyset$
  - BSML^{+}: BSML + global pragmatic enrichment
  - $\text{BSML}^{\emptyset}$: BSML without NE (= classical logic)
  - BSML^{lex}: BSML + local pragmatic enrichments (lexicalizations)

Global suspension of neglect-zero effects: BSML$^{\emptyset}$

- Despite their cognitive cost, zero-models are not always neglected.
- In logico-mathematical reasonings, neglect-zero effects are globally suspended:

(19) A. Therefore, A or B.

(20) A or B. Not A. Therefore, B.

(21) If A then B. Therefore, if not B then not A.

- Global suspension modeled in BSML by NE-free fragment $\mapsto$ BSML$^{\emptyset}$

		BSML$^{\emptyset}$	BSML^{+}
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	-	+
Addition	$\alpha \models \alpha \vee \beta$	+	-
Disjunctive syllogism	$(\alpha \vee \beta) \wedge \neg\alpha \models \beta$	+	-
Contraposition	$\alpha \models \beta \;\Rightarrow\; \neg\beta \models \neg\alpha$	+	-

- In BSML$^{\emptyset}$ (= classical logic), $\emptyset$ plays an essential role:

> The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (Alfred North Whitehead, quoted by Nieder 2016)

Neglect-zero effects: local suspension?

- Suppose $[\ ]^+$ were a grammatical operation which can optionally apply [just like EXH in localist accounts of scalar implicatures, Chierchia et al 2008)
- We would predict $\Diamond([\alpha]^+ \vee \beta)$ and $\Diamond(\alpha \vee [\beta]^+)$ as possible readings of 'You may do α or β'.
- This prediction does not seem to be correct. Mom's reaction in the following dialogue is incoherent:

(22) MOM: You may do your homework or go to the beach.
SON: Ok, then I go to the beach.
MOM: No I only meant that you may do your homework.

- Conclusion: $[\ ]^+$ is not an optional grammatical operation

Lexically triggered neglect-zero enrichments: BSML^{lex}

- Potential problem for a view which only allows global suspension of neglect-zero effects:

(23) a. I may do A or B or I may do C. I may not do C. Therefore, I may do A and I may do B.
b. $(\Diamond[\alpha \vee \beta]^+ \vee \Diamond\gamma) \wedge \neg\Diamond\gamma \models \Diamond\alpha \wedge \Diamond\beta$

- Conjecture: modal verbs trigger pragmatic enrichment in their prejacents as part of their lexical meaning $\mapsto$ BSML^{lex}

(24) a. I may do A or B or I may do C. I may not do C. Therefore, I may do A and I may do B.
b. $(\Diamond[\alpha \vee \beta]^+ \vee \Diamond[\gamma]^+) \wedge \neg\Diamond[\gamma]^+ \models \Diamond[\alpha]^+ \wedge \Diamond[\beta]^+$

- BSML^{lex} predicts a contrast between positive FC (valid) vs negative FC & modal disjunction (not valid), which gives in combination with BSML* a better match with experimental findings:

			BSML^{lex}	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**s**	$+$	$+$
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	$+$
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**w**	-	$+$

What about overt FC cancellations?

- Overt FC cancellation:

(25) You may either eat the cake or the ice-cream, I don't know which
$\not\rightsquigarrow$ You may eat the cake

- Prediction of BSML^{lex}: all cases of overt FC cancellations involve a wide scope configuration:
  1. Narrow scope FC: $\Diamond[\alpha \vee \beta]^+ \models \Diamond[\alpha]^+ \wedge \Diamond[\beta]^+$
  2. Wide scope FC: $\Diamond[\alpha]^+ \vee \Diamond[\beta]^+ \not\models \Diamond[\alpha]^+ \wedge \Diamond[\beta]^+$
- Sluicing in (26) arguably triggers wide scope (Fusco 2018):

(26) You may either eat the cake or the ice-cream, I don't know which (you may eat). [wide, –fc]

- Wide scope configuration also required for (27) (Kaufmann 2016):

(27) You may either eat the cake or the ice-cream, it depends on what John has taken. [wide, –fc]

- Wide scope FC captured as weak/global neglect-zero effect:
  - BSML*: $\Diamond\alpha \vee \Diamond\beta \models_{BSML^*} \Diamond\alpha \wedge \Diamond\beta$ [if R is indisputable]

The resulting picture: pluralism

- A pluralism of interpretation strategies & reasoning styles people may adopt in different circumstances:
  1. $\text{BSML}^{\emptyset}$: modelling logical-mathematical reasoning where neglect-zero effects are obviated;
  2. BSML^{lex}: modelling local neglect-zero effects due to lexicalisations in modal verbs;
  3. BSML^{*}: modelling global (and weak) neglect-zero effects.
- Experimentally testable predictions arising from these conjectures

			$\text{BSML}^{\emptyset}$	BSML^{lex}	BSML^{*}
NS FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	s	-	+	+
Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	s	+	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	w	-	-	+
Modal disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	w	-	-	+
WS FC	$\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	?	-	-	+

Table: Comparison $\text{BSML}^{\emptyset}$, BSML^{lex} and BSML^{*}.

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but, when applied to FC, empirically inadequate
- My proposal: Neglect-zero effects in BSML
  - Classical logic (NE-free fragment) + neglect-zero (NE) $\Rightarrow$ FC and related inferences
  - Suspension and locality of neglect-zero effects:
    - Pluralism: $\text{BSML}^{\emptyset}$ vs BSML^{lex} vs BSML^{*}
- Related (future) research:
  - Logic: proof theory (Anttila, Yang, MA); bimodal perspective (Baltag, van Benthem, Bezhanishvili, MA);
  - Language: FC cancellations (Pinton 2021, Hui 2021); modified numerals (MA & van Ormondt); indefinites (MA & Degano); more on 'neglect $\emptyset$', its cognitive plausibility and BSML^{lex} vs BSML^{*}.