# Neglect-zero and no-split: cognitive biases at the semantic-pragmatic interface

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# N$\emptyset$thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic/cognitive factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (and **no-split**) as crucial pragmatic/cognitive factors
- Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind


## Nihil team

MA, Anttila, Knudstorp, Degano, **Klochowicz**, Ramotowska, **Sbardolini**

# Non-classical inferences

## Free choice (FC)

(1) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

## Ignorance

(4) The prize is in the attic *or* in the garden $\rightsquigarrow$ speaker doesn't know where [Grice 1989]

(5) ? I have two *or* three children.

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC inferences analysed as conversational implicatures; grammatical implicatures; semantic entailments; . . .

# Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ scalar implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers create mental structures representing reality[1] and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets [Nieder 2016, Bott et al, 2019]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

# Novel hypothesis: neglect-zero

## Illustrations

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]; [□, □, □]

- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al., 2019]
  - existential import & connexive principles operative in Aristotelian logic (*every A is B* $\Rightarrow$ *some A is B*; *not (if not A then A)*) [MA, 2024]
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences

# Novel hypothesis: neglect-zero

## Illustrations

(8) It is raining.

- a. Verifier: [//// //// ////]
- b. Falsifier: [☼☼☼]
- c. Zero-models: none

(9) It is snowing.

- a. Verifier: [❄❄❄]
- b. Falsifier: [☼☼☼]; [//// //// ////]; ...
- c. Zero-models: none

(10) It is raining or snowing.

- a. Verifier: [//// //// //// | ❄❄❄]
- b. Falsifier: [☼☼☼]
- c. **Zero-models**: [//// //// ////]; [❄❄❄]

- Two models in (10-c) are **zero-models** because they verify the sentence by virtue of an empty witness for one of the disjuncts
- Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded

# Novel hypothesis: no-split

## A closer look at the disjunctive case

(11) It is raining or snowing.

a. Verifier: [//// //// //// | ❄❄❄] [⇐ “split” state]

b. Falsifier: [☼☼☼]

c. Zero-models: [//// //// ////]; [❄❄❄]

- The “split” verifier in (11-a) involves the entertainment of two alternatives, also arguably a cognitively difficult operation
- We conjecture that the ability to split states is acquired late

  ↦ NO-SPLIT HYPOTHESIS
- The combination of neglect-zero and no-split can explain non-classical inferences observed in pre-school children

# No-split and the acquisition of ‘or’

- Basic data: some pre-school children interpret *or* as *and* [e.g., Singh *et al* 2016, Cochard 2023, Bleotu *et al* 2024]:

(12) The boy is holding an apple or a banana = The boy is holding an apple and a banana $(\alpha \vee \beta) = (\alpha \wedge \beta)$

(13) Every boy is holding an apple or a banana = Every boy is holding an apple and a banana $\forall x(\alpha \vee \beta) = \forall x(\alpha \wedge \beta)$

(14) Liz can buy a croissant or a donut = Liz can buy a croissant and a donut $\Diamond(\alpha \vee \beta) = \Diamond(\alpha \wedge \beta)$

- Two different explanations:
  - Singh et al: derive $\alpha \wedge \beta$ from $\alpha \vee \beta$ as a scalar implicature using exh-ALT $= \{\alpha \wedge \neg\beta, \beta \wedge \neg\alpha\}$ [or, alternatively, by innocent inclusion] $\mapsto$ children can compute scalar implicatures and can exhaustify alternatives, but don’t have access to lexical alternatives
  - Nihil: beside neglecting zero-models, children further lack the ability to split states, i.e. have difficulties in engaging with alternative epistemic possibilities, in picturing different ways the world might be.

# BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]

## Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

## Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Dekker93; Groenendijk+96; Ciardelli+19]
- Assertion & rejection conditions modelled rather than truth

$$M, s \models \phi, \text{ ``}\phi \text{ is assertable in } s\text{''}, \quad \text{with } s \subseteq W$$
$$M, s \mathrel{=\mkern-3mu|} \phi, \text{ ``}\phi \text{ is rejectable in } s\text{''}, \quad \text{with } s \subseteq W$$

- Neglect-zero tendency modelled by $\mathrm{NE}$ [Yang & Väänänen 2017]
- $\mathrm{BSML}^{\mathrm{F}}$: No-split modelled via a flattening operator $\mathrm{F}$ [Punčochář 2024]

# $\text{BSML}^{\text{F}}$: Classical Modal Logic + $\text{NE}$ + $\text{F}$

## Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \text{NE} \mid \text{F}$$

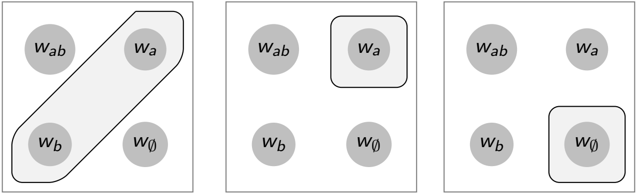
## Bilateral team semantics

$[M = \langle W, R, V\rangle \;\&\; s, t, t' \subseteq W]$

$$
\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w, p) = 1 \\
M, s \mathrel{=\!\!|} p & \text{iff} & \text{for all } w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \mathrel{=\!\!|} \phi \\
M, s \mathrel{=\!\!|} \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi \\
M, s \mathrel{=\!\!|} \phi \vee \psi & \text{iff} & M, s \mathrel{=\!\!|} \phi \;\&\; M, s \mathrel{=\!\!|} \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \;\&\; M, s \models \psi \\
M, s \mathrel{=\!\!|} \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \mathrel{=\!\!|} \phi \;\&\; M, t' \mathrel{=\!\!|} \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi \\
M, s \mathrel{=\!\!|} \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \mathrel{=\!\!|} \phi \\
M, s \models \text{NE} & \text{iff} & s \neq \emptyset \qquad\qquad [\text{where } R[w] = \{v \in W \mid wRv\}] \\
M, s \mathrel{=\!\!|} \text{NE} & \text{iff} & s = \emptyset \\
M, s \models \text{F}\phi & \text{iff} & \text{for all } w \in s : M, \{w\} \models \phi \\
M, s \mathrel{=\!\!|} \text{F}\phi & \text{iff} & \text{for all } w \in s : M, \{w\} \mathrel{=\!\!|} \phi
\end{array}
$$

# Neglect-zero effects in BSML: split disjunction

- A state $s$ supports a **disjunction** $\phi \vee \psi$ iff $s$ is the union of two substates, each supporting one of the disjuncts



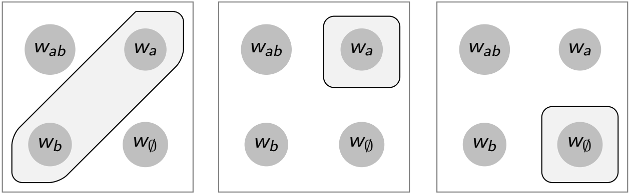

(a) Verifier (b) **Zero-model** (c) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ & $M, \emptyset \models b$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\;]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\;]^+$ defined using $\mathrm{NE}$ ($s \models \mathrm{NE}$ iff $s \neq \emptyset$), which models neglect-zero in the logic

# Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts



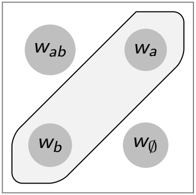

(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\Dashv [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities

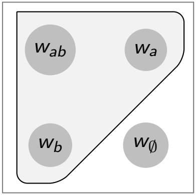
(15) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions [MA 2022]
  - $\mapsto$ we derive FC and related effects (for $[\ ]^+$-enriched formulas);
  - $\mapsto$ $[\ ]^+$-enrichment vacuous under single negation.

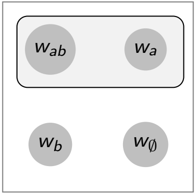
# More no-zero verifiers for enriched disjunction



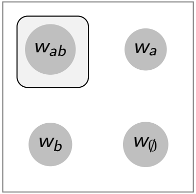

(d) no-zero & scalar $\models \neg(a \wedge b)$




(e) no-z & non-scalar $\not\models \neg(a \wedge b)$




(f) no-z, non-scalar & no-uncertain $\not\models \neg\Box a$

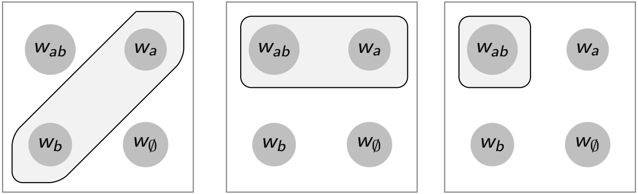



(g) no-z, non-scalar, no-uncert. & no-split

Figure: Models for enriched $[a \vee b]^+$.

# Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \vee b$:




(a) scalar (b) no-uncertain (c) no-split

- Two components of full ignorance ('speaker doesn't know which'):[2]

(16) It is raining or it is snowing ($\alpha \vee \beta$) $\rightsquigarrow$

a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$ )

- **Fact:** Only possibility derived as neglect-zero effect:
  - $\{w_{ab}, w_a\} \models \Diamond_e a \wedge \Diamond_e b$, but $\not\models \neg\Box_e a$
  - $\{w_{ab}, w_a\}$: a no-zero model supporting possibility but neither uncertainty nor scalar implicature ($\not\models \neg(a \wedge b)$)

[2] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. SuB & XPRAG, 2023.

# Two derivations of full ignorance

1. Neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (17) $\alpha \vee \beta$ ASSERTION

   (18) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (19) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (20) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Nihil derivation

   (i) Possibility derived as neglect-zero effect

   (21) $\alpha \vee \beta$ ASSERTION

   (22) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (23) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (24) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

# Neglect-zero effects in BSML: possibility vs uncertainty

## Comparison with competing accounts

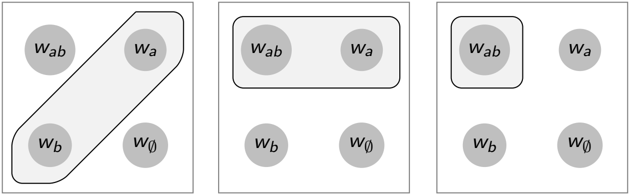
- Neo-Gricean vs Nihil predictions
  - Neo-Gricean: No possibility without uncertainty
  - Nihil: Possibility derived independently from uncertainty

## Experimental study

- Experimental findings in agreement with Nihil predictions
  [Degano *et al* 2023]
  - Using adapted mystery box paradigm, compared conditions in which
    - both uncertainty and possibility are false [**zero-model**]
    - uncertainty false but possibility true [no-zero, no-uncertain model]
  - Less acceptance when possibility is false (95% vs **44%**)
- Evidence that possibility can arise without uncertainty

# Neglect-zero and no-split

- More no-zero verifiers for $a \vee b$:



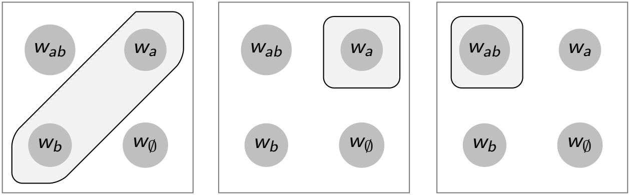

(d) scalar (e) no-uncertain (f) no-split

- $\{w_{ab}\}$ is a no-split verifier for the disjunction: no alternatives entertained;
- Conjecture: only no-split verifiers accessible to 'conjunctive' pre-school children [Klochowicz, Sbardolini, MA, 2024]
- Combination of no-split and no-zero gives us conjunctive *or*
- Implementation: uses flattening operator $\mathrm{F}$

$$M, s \models \mathrm{F}\phi \text{ iff for all } w \in s : M, \{w\} \models \phi$$

Flattening $\mapsto$ formulas always interpreted wrt to singleton states

# Illustration




(g) no-zero & split (h) zero & no-split (i) no-zero & no-split

Figure: Combination of no-split and no-zero gives us conjunctive *or*

(25) It is raining or snowing.

a. No-zero & split: [ | ] [adult-like]

b. Zero and no-split: [ ] [logician]

c. No-zero & no-split: [ + ] ['conjunctive' children]

# No-split: some predictions

(26)
a. $[\mathrm{F}(\alpha \vee \beta)]^{+/*} \equiv \alpha \wedge \beta$
b. $[\forall x \mathrm{F}(\alpha \vee \beta)]^{+/*} \equiv \forall x(\alpha \wedge \beta)$
c. $[\Diamond \mathrm{F}(\alpha \vee \beta)]^{+/*} \equiv \Diamond(\alpha \wedge \beta)$ $[\not\equiv \Box(\alpha \wedge \beta)]$
d. $[\neg \mathrm{F}(\alpha \vee \beta)]^{+/*} \equiv \neg\alpha \wedge \neg\beta$
e. $[\neg \mathrm{F}(\alpha \wedge \beta)]^{*} \equiv \neg\alpha \wedge \neg\beta$, but $[\neg \mathrm{F}(\alpha \wedge \beta)]^{+} \not\equiv \neg\alpha \wedge \neg\beta$

- Two ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function $[\,]^{+}$ defined in terms of NE $\mapsto$ BSML$^{+}$
  - Model-theoretically, by ruling out $\emptyset$ from the set of possible states $\mapsto$ BSML$^{*}$
- Both implementations derive:
  - $\mapsto$ FC effects (narrow and wide scope FC, dual prohibition, etc);
  - $\mapsto$ conjunctive *or* in combination with flattening (26-a-d).
- But only BSML$^{*}$ predicts
  - Negative FC: $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$ [Marty et al]
  - Homogeneity effects in combination with $\mathrm{F}$ (26-e) [Sbardolini23]
- Only in BSML$^{+}$, $\emptyset$ is part of the building blocks (natural to assume BSML$^{*}$ for "conjunctive" children who plausibly do not access $\emptyset$)

# Two views

- Two explanations of conjunctive ‘or’ in pre-school children:
  - Grammatical view: conjunctive children can compute implicatures but do not have access to scalar alternatives (or $<$ and);
  - Nihil: conjunctive behaviour derives from the combination of two cognitive bias: no-zero and no-split.

| | **conjunctive** *or* | **inclusive** *or* | **exclusive** *or* |
|---|---|---|---|
| Grammatical | exh-alt [✓] | exh-alt [no] | scalar-alt [✓] |
| Nihil | zero [no] & split [no] | split [✓] (or zero [✓] ) | split [✓] & scalar reasoning [✓] |

- Two different acquisition patterns:
  - Grammatical view:
    inclusive *or* $<$ conjunctive *or* $<$ exclusive *or*
  - Nihil:
    conjunctive *or* $<$ inclusive *or* $<$ exclusive *or*

# Conclusions

- FC and related inferences: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: stronger meanings consequences of cognitive biases
  - FC and ignorance as neglect-zero effects

    Literal meanings (NE-free fragment) + cognitive factors (NE) $\Rightarrow$ FC & possibility inferences

  - Conjunctive *or* as no-zero + no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in $\text{BSML}^{\text{F}}$ (a team-based modal logic)

# Collaborators & related (future) research

## Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Yang, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); Aristotelian logic in $\text{qBSML}_{\rightarrow}$ (MA);...

## Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Ramotowska, Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); acquisition (Sbardolini, Klochowicz); ...

THANK YOU![3]

[3]This work is supported by the NWO OC project *Nothing is Logical (NihiL)* (grant no 406.21.CTW.023)